International Portfolio Choice and the Economic Value of Jump Timing

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Abstract

This paper investigates the frequency of jumps in both stock market returns and volatility. It is well known that jump events induce less volatility persistence than the normal innovations. If jump is rare and unpredictable, then controlling for jumps should provide more accurate measure and forecast of volatility and correlation. Using returns on 34 stock markets we demonstrate that accounting for jumps adds substantial economic value to asset allocation strategy. A dynamic volatility-jump-timing strategy outperforms both a dynamic volatility timing strategy and a static strategy for asset allocation. Our results are robust to the choice of jump estimation method.

JEL codes: C32, F30, F31, G11, G12, G15

Keywords: Economic value, volatility timing, jumps, asset allocation

1. Introduction

It is well established in the literature that international stock return volatility and covariance are time varying and, to a certain degree, predictable. However although time-varying volatility models can produce some of the non-normality in returns they fail to capture extreme price movements. Recent work highlights that returns and volatility can be characterised by jumps which are more successful in capturing such extremes (Jorion, 1988; Bates, 2000; Pan, 2002; Eraker et al., 2003; Eraker, 2004; Broadie et al., 2007). Furthermore such jumps typically occur at the same time across markets (Das and Uppal, 2004) having a large impact on correlations and covariances. Here, we argue that jumps, like outliers, are rare, unpredictable and belong to another distribution. Consequently, jumps, if not identified and removed, can have a large impact on the estimates of conditional variance and covariance of returns. Our focus in this paper is to highlight the importance of jumps for accurate estimation of volatility and to evaluate the implications for asset allocation strategies based on volatility timing.

There are several recent studies that clearly demonstrate the economic benefits of predicting and timing volatility (e.g. Fleming, Kirby and Ostdiek, 2001; 2003; Johannes, Polson and Stroud, 2002; Marquering and Verbeek 2004; Della Corte, Sarno and Tsiakias, 2008). Fleming Kirby and Ostdiek (2001) are the first to show that modelling time-varying volatility/covariance has economic value to risk-adverse investors. More specifically, that a dynamic portfolio strategy which uses time-varying covariance (volatility timing) significantly outperforms a static portfolio strategy based on constant expected returns, volatilities and covariance. Fleming, Kirby and Ostdiek (2003) highlight the importance of adopting appropriate measures of conditional volatility on which to base any volatility timing strategy. Comparing alternative models for volatility, they find that those models that are based on intraday returns provide more precise estimates and substantial economic value.

Further support for volatility timing is provided by Johannes, Polson and Stroud (2002) who indicate that such strategies are relatively immune from the risk estimation which besets strategies based on time-varying expected returns. Moreover, Marquering and Verbeek (2004) examine the use of predictive macroeconomic and financial variables to time returns and show that the economic value of trading strategies that employ timing in both returns and volatility is superior to that of strategies that employ market timing in returns only. These timing benefits are not restricted to equity markets; Della Corte, Sarno and Tsiakias (2008) provide evidence on the value of volatility timing in the context of exchange rate models.

The importance of jumps and higher moments has also been noted in the literature. As identified by Das and Uppal (2004), jumps tend to occur at similar times across markets. While they show the cost of ignoring such systemic jumps is only small in terms of the reduction in the gains from international diversification, it is large for highly levered portfolios. Ang and Bekaert (2002) and Guidolin and Timmermann (2008) highlight how regime switching behavior in mean and variance can capture the distributional properties of returns and have significant impact on the international asset allocation problem. Guidolin and Timmermann (2008) focus, in addition, on the importance of higher moments (skewness and kurtosis) demonstrating how taking account of these moments significantly improves asset allocation. This distributional timing is considered by Jondeau and Rockinger (2007) who demonstrate that such strategies increase economic value beyond that offered by a dynamic mean-variance approach. Here, we argue that jumps could potentially bias these high order statistics.

Jumps that occur in two or more markets simultaneously can significantly bias the correlation. If jumps are unpredictable and rare, the assumption here is that such co-jump is also unpredictable.

In this paper, we investigate the presence of jumps in international equity returns and volatility, and examine if the removal of these jumps can help time volatility better. Our work is closely related to Maheu and McCurdy (2004) who find evidence of superior volatility forecast with a mixture-GARCH-Jump-Intensity model. Separating jumps from diffusion allows the volatility persistence of both components to be estimated more accurately since it is well know that the 'normal' diffusion tends to have stronger volatility persistence than the jump part. Unlike Wu (2003) who studies asset allocation with only one risky asset with a preference function that is sensitive to jumps, we consider the asset allocation problem for an U.S. investor who takes equity positions in 34 international markets. To estimate the jump processes in both returns and volatility we adopt the Markov Chain Monte Carlo (MCMC) approach advocated by Raggi (2005). We find that all stock return series are characterised by jumps and that jumps do occur simultaneously in different markets. To model the conditional volatility and correlation structure of the portfolio we use the AG-DCC-GARCH model of Cappiello, Engle and Shepard (2006).

Consistent with the established literature we show, for our sample, that a dynamic asset allocation strategy based on weekly volatility timing has significant economic value over a static investment strategy. However, further improvements are made with a strategy that cleans returns and volatility for jumps. Based on three common performance measures we demonstrate consistently that there are clear economic benefits to volatility and jump timing in international equity asset allocation.

Moreover, we show that our findings are robust to the choice of jump estimation method. The superiority of the volatility and jump timing strategy holds in 11 out of 12 years in our sample. The year of exception is 1998. In 1998, several of the Asian markets experienced persistence in jumps during the Asian financial crisis. If we define a systemic event as one when five or more stock markets co-jumps, then in 1998 there are all together seven such systematic event days.

The remainder of the paper is as follows. Section 2 describes the methodology utilised, providing details of the jump, volatility, correlation and covariance estimation and the measurement of economic value. Section 3 discusses the data and descriptive statistics while section 4 provides the analysis of the empirical results. Section 5 concludes.

2. Methodology

This section discusses the framework we use to evaluate the impact of timing volatility and jumps on the performance of one-week-horizon dynamic asset allocation strategies. We employ the standard mean-variance optimization to implement the dynamic asset allocation strategies and apply three common performance measures; the Sharpe ratio, the performance-fee measure, and break-even transaction costs to measure the economic value of these strategies. We use an efficient Markov Chain Monte Carlo (MCMC) procedure proposed by Raggi (2005) to estimate jumps in both price and volatility. Finally, we examine whether timing jumps has economic value by comparing the performance of the dynamic strategy based on data that has jumps removed with other strategies that do not remove jumps from the raw data.

2.1 Dynamic Portfolio Choice in Mean-Variance Framework

Instead of using mean, variance, skewness and kurtosis to construct portfolio and a high moment based preference function to evaluate the portfolio performance, we have decided to adopt the simpler traditional mean-variance framework. Mean-variance optimization is still the most common approach used in the industry for selecting portfolios largely due to its simplicity and stability.[1] Further it provides a natural framework to compare strategies which emphasize predictability in the mean and variance of returns. Other high moment based strategies could be considered separately as "added-on" features to our jump-volatility timing strategy. In the mean-variance framework, the investor is faced with a trade-off between the portfolio expected return, and the risk, measured by the variance of the portfolio returns. The investor can either choose to minimize variance subject to achieving a particular targeted expected return (the minimum risk objective), or maximize portfolio return given a targeted level of risk (the maximum return objective). Implementation of mean-variance optimization requires the forecasts of returns and conditional variance-covariance matrix of returns over the intended investment horizon.

Let r_{t+1} denote a $N \times 1$ vector of actual risky asset returns in period t+1 (N=34 here); $\mu = E[r_{t+1}]$ is the unconditional expected value of r_{t+1};[2] and $\Omega_{t+1} = E_t[(r_{t+1} - \mu)(r_{t+1} - \mu)']$ is the forecasted variance-covariance matrix of r_{t+1}. Consider an investor with one-week investment horizon (t=1 week) who wants to

1 The evidence for this is the popularity of the use of the Sharpe ratio in evaluating performance in practice. Tail thickness (higher moments) has a much greater direct impact on risk management than on investment decisions.

2 Section D.2 provides details of the calculation of the forecast of the unconditional expected value of returns.

implement the target-return-minimum-risk strategy. For each period *t*, the investor solves the following quadratic program:

$$\min_{w_t}\{w'_t\, \Omega_{t+1} w_t\}$$

$$\text{s.t. } w'_t\, \mu + (1 - w'_t 1) r_f = \mu_p^* \tag{1}$$

where w_t is $N \times 1$ vector of portfolio weights on risky assets, r_f is the return on a riskless asset (assumed to be a constant 6% per annum), and μ_p^* is the portfolio target expected return.

A similar analysis can also be applied if the objective is to maximize the portfolio expected return subject to achieving a particular portfolio variance. In this case, the investor solves the following quadratic program at time *t*:

$$\max_{w_t}\{\mu_p^* = w'_t\, \mu + (1 - w'_t 1) r_f\}$$

$$\text{s.t. } w'_t\, \Omega_{t+1} w_t = (\sigma_p^*)^2 \tag{2}$$

where σ_p^* is the portfolio target level of volatility.

To calculate the portfolio weights of the optimal portfolio in the mean-variance framework, a forecast of the covariance matrix is required. Several suitable estimation methods have been developed in the literature.[3] We employ the recently developed asymmetric dynamic conditional correlation GARCH model (AG-DCC-GARCH) of Cappiello, Engle and Sheppard (2006) to calculate conditional covariance matrix. The advantage of this model is that it explicitly captures the asymmetric response of

[3] See, for example, the ad hoc rolling estimators (Officer, 1973; Fama and MacBeth, 1973); the weighted rolling average of the squares and cross product of past return innovations (Forster and Nelson, 1996; Fleming, Kirby and Ostdiek, 2001); and the ARCH/GARCH approach (Engle, 1982; Bollerslev, 1986).

conditional volatilities, correlations and covariances to negative returns. The importance of allowing for asymmetric effects is noted by De Goeij and Marquering (2004) who find that investors can benefit from tactical asset allocation when asymmetric effects in covariances are taken into account.

Given that the presence of jumps in price and volatility could lead to inaccurate estimates of covariance, we remove jumps from returns and re-estimate the conditional variance-covariance matrix. Since our main purpose is to investigate if the removal of jumps could improve variance-covariance estimation and hence a better portfolio choice, we compare the performance of different asset allocation strategies: the dynamic jump-removed strategy and other static and dynamic strategies without removing jumps. The performance of each of these strategies is evaluated based on realised portfolio returns (i.e. actual returns, not jump-removed returns). One question that follows immediately is, 'How should one remove jumps?'. For each jump detected, we replace the return with the maximum (minimum) return in the previous 8 weeks. This seems to be a suitable method since it allows the data to reflect the recent volatility level.[4]

2.2 Modelling Jumps in Price and Volatility

We employ an efficient Markov Chain Monte Carlo (MCMC) procedure recently proposed by Raggi (2005) to estimate affine jump diffusion models. The framework adopted is the following stochastic volatility model in which equity log-prices Y_t and the volatility process V_t are given by: (Duffie et al.,2000)

$$dY_t = \left(\phi - \frac{1}{2}V_t\right)dt + \sqrt{V_t}dW_t^y + dJ_t^y \qquad (3)$$

[4] We have experimented with several other 'partial winsorisation' methods such as replacing the jump return by either the 5 × sample medium or 1/3 of the jump return itself, but find the results are all qualitatively similar.

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^v + dJ_t^v \quad (4)$$

$$dJ_t^i = H_t^i dN_t^i, \qquad i = \{y, v\} \quad (5)$$

where ϕ is the drift of the log prices; W_t^y and W_t^v are two correlated Brownian motions and the correlation parameter, $\rho = E[W_t^y W_t^v]$, is often interpreted as the leverage effect and induces a relationship between volatilities and returns. The parameters κ, θ and σ_v are, respectively, the mean reversion, the long run mean and the standard error of the volatility process.[5] For $i = \{y, v\}$, the jump term, dJ_t^i, has a jump-size component H_t^i, and N_t^i is the number of arrivals of a Poisson process in the interval $(0, t)$. In this paper, the jump intensities $\lambda_v = \lambda_y$ are constant over time, and the model allows both returns and volatility to jump as follows:

$$H_t^v \sim \exp(\phi_v),$$

$$H_t^y \mid H_t^v \sim N(\phi_y + \rho_j H_t^v, \sigma_y^2).$$

As shown above, jumps to volatility and returns are driven by the same Poisson process ($N_t^y = N_t^v$). The jump sizes, H_t^y and H_t^v, are different but are correlated with the coefficient ρ_j. For this reason, this model is called stochastic volatility with correlated jumps (SVCJ). The model assumes that jumps in price will simultaneously impact on both returns and volatility, and if the leverage effect parameter, ρ, is negative, a market crash would lead to a dramatic increase in volatility increase.

Markov Chain Monte Carlo (MCMC) estimation and inference is performed using Equations (3) and (4). Since volatilities and jumps are unobserved, they are

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[5] To avoid problems with the potential negativity of volatilities, we consider the logarithmic transform log(Vt).

considered as missing data in the MCMC setting where the target distribution is $\psi(\theta, V, H^v, H^y \mid Y)$ and $Y = \{Y_1, ..., Y_T\}$ is the completed vector. More details on the procedure can be found in the Appendix A and in Raggi (2005).

2.3 Estimating the Conditional Covariance Matrix

To estimate and forecast the conditional covariance matrix, we employ the recently developed asymmetric dynamic conditional correlation GARCH model (AG-DCC-GARCH) of Cappiello, Engle and Sheppard (2006). This is a generalisation of the DCC-MVGARCH model of Engle (2002) to capture the conditional asymmetries in correlation. We estimate the DCC-MVGARCH using a two-stage procedure. In the first stage, we model asset returns using univariate asymmetric GARCH specifications generating standardized residuals and volatilities for each series. The second stage uses these standardized residuals to estimate the coefficients governing dynamic correlation.

Let e_t denote the, time t, n x 1 vector of return innovations (residuals); e_t is assumed to be conditionally normal with mean zero and covariance matrix $\mathbf{H}_t$:

$$\boldsymbol{e_t} \left| \Phi_{t-1} \sim \boldsymbol{N}\left(0, \mathbf{H}_t\right) \right. \tag{6}$$

$$h_{i,t} = \omega + \alpha e_{i,t-1}^2 + \gamma I[e_{i,t-1} < 0] e_{i,t-1}^2 + \beta h_{i,t-1} \qquad \text{(GJR-GARCH)} \tag{7}$$

$$\varepsilon_{i,t} = e_{i,t} \Big/ \sqrt{h_{i,t}} \tag{8}$$

where Φ_{t-1} represents the information set at time t-1, and the conditional covariance matrix $\mathbf{H}_t$ can be decomposed as follows:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \tag{9}$$

where $\mathbf{D}_t = diag\left\{\sqrt{h_{i,t}}\right\}$ is the $n \times n$ diagonal matrix of time-varying standard deviations from univatiate GARCH models with $\sqrt{h_{i,t}}$ on the ith diagonal, and $\mathbf{R}_t$ is the $n \times n$ time-varying correlation matrix, containing conditional correlations. The proposed dynamic correlation structure is:

$$\mathbf{R}_t = diag(\mathbf{Q}_t)^{-1}\mathbf{Q}_t diag(\mathbf{Q}_t)^{-1} \quad (10)$$

$$\mathbf{Q}_t = (\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}) + \mathbf{A}'\varepsilon_{t-1}\varepsilon'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{Q}_{t-1}\mathbf{B} + \mathbf{G}'\eta_{t-1}\eta'_{t-1}\mathbf{G} \quad (11)$$

where $diag(\mathbf{Q}_t) = \left\lfloor \sqrt{q_{i,i,t}} \right\rfloor$ is a diagonal matrix containing the square root of the diagonal elements of $\mathbf{Q}_t$, **A**, **B** and **G** are $n \times n$ parameter matrices, $\varepsilon_{i,t} = e_{i,t} / \sqrt{h_{i,t}}$ is the standardized residuals, $\overline{\mathbf{Q}} = E[\varepsilon_t \varepsilon'_t] = T^{-1}\sum_{t=1}^{T}\varepsilon_t\varepsilon'_t$ is the unconditional correlation matrix of e_t, and $\overline{\mathbf{N}} = E[\eta_t\eta'_t] = T^{-1}\sum_{t=1}^{T}\eta_t\eta'_t$, with $\eta_{i,t} = I_{[\varepsilon_{i,t}<0]} \circ \varepsilon_{i,t}$, where $I_{[\varepsilon_{i,t}<0]}$ is the indicator function which takes on value 1 if $\varepsilon_{i,t} < 0$ and 0 otherwise, and "$\circ$" denotes the Hadamard product. This term captures the conditional asymmetries in correlations.[6] It is clear from equation (11) that $\mathbf{Q}_t$ will be positive-definite if $(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G})$ is positive definite. The AG-DCC model is estimated using quasi-maximum likelihood (QMLE).

The covariance between asset i and asset j can be obtained (from (7) and (10)):

$$Cov_{i,j,t} = \rho_{i,j,t}\sqrt{h_{i,t}}\sqrt{h_{j,t}} \quad (12)$$

Finally, we use one-step-ahead forecast of $h_{i,t}$ from (7) in (12) to forecast the variance-covariance of returns. This method allows the parameter estimates to be updated sequentially each week throughout the out-of-sample period.

[6] The generalized DCC (G-DCC) model is a special case of AG-DCC when **G**=0.

2.4 Measuring the Economic Value of Volatility and Jump Timing

To measure the economic value of jump timing we compare the performance of the competing investment strategies employing three different performance measures: the Sharpe ratio, the utility performance-fee measure, and the break-even transaction cost.

2.4.1 Sharpe ratio

The weekly Sharpe ratio, SR_t, is calculated using the realized portfolio return, $r_{p,t}$, and realized portfolio standard deviation (from realized return), $\sigma_{p,t}$:

$$SR_t = (\mu_{p,t} - r_f) / \sigma_{p,t} \tag{13}$$

The Sharpe ratio measures the excess portfolio return (over risk-free rate) per unit portfolio risk (volatility). A portfolio performs better than another if it has a larger Sharpe ratio. However, although this relatively simple measure of "bang for buck" is an industry standard, Marquering and Verbeek, (2004) and Han (2006) note that the Sharpe ratio can severely underestimate the performance of dynamic strategies, since the sample standard deviation of the realized portfolio returns tends to overestimate the risk an investor faces. Moreover, the simple Sharpe ratio provides no indication of the economic value of dynamic strategies over a static strategy. Hence we augment our analysis with two further performance measures.

2.4.2 Utility performance-fee measure

We use a generalisation of West, Edison and Cho's (1993) methodology to rank the performance of competing models. The method is based on mean-variance analysis and quadratic utility. The investor's realized utility in period t can be written as:

$$U(W_t) = W_{t-1} r_{p,t} - \frac{aW_{t-1}^2}{2} r_{p,t}^2 \tag{14}$$

where W_t is the investor's wealth at period t, a is the investor's level of absolute risk aversion, and

$$r_{p,t} = (1 - w'_{t-1} 1) r_f + w'_t \, r_t \tag{15}$$

is the period t return on his portfolio. To facilitate comparison across portfolios, we hold aW_t constant, which is equivalent to setting the investor's relative risk aversion, $\gamma_t = aW_t / (1 - aW_t)$, equal to some fixed value γ. With relative risk aversion being held constant, we can use the average realized utility, $\overline{U}(.)$, to consistently estimate the expected utility generated by a given level of initial wealth W_0. Specifically, we have

$$\overline{U}(\cdot) = W_0 \sum_{t=1}^{T} \left(r_{p,t} - \frac{\gamma}{2(1+\gamma)} r_{p,t}^2 \right) \tag{16}$$

Following Fleming, Kirby and Ostdiek (2001), we consider a performance-fee measure Δ for each of the dynamic strategies. The amount Δ can be interpreted as the maximum weekly management fee such that an investor would be indifferent between the static strategy and the dynamic strategy.

$$\sum_{t=1}^{T} \left((r_{d,t} - \Delta) - \frac{\gamma}{2(1+\gamma)} (r_{d,t} - \Delta)^2 \right) = \sum_{t=1}^{T} \left(r_{s,t} - \frac{\gamma}{2(1+\gamma)} r_{s,t}^2 \right) \tag{17}$$

where $r_{d,t}$ and $r_{s,t}$ denote the returns for dynamic and static strategies, respectively. The estimates of Δ are reported as annualized fees in basis points using two different values of γ, i.e. $\gamma = 1$ and $\gamma = 10$. The emphasis here is on the relative performance of the competing strategies. The assumptions of quadratic utility and risk aversion level are not key issues since they have the same impact on all strategies.

2.4.3 Break-even transaction costs

Accounting for transaction costs is important in comparing different trading strategies. However, there is no consensus, in the literature, on an appropriate figure for transaction cost. Several studies use the proportional transaction costs (Stoll and Whaley, 1983; Constantinides, 1986; Bhardwaj and Brooks, 1992; Balduzzi and Lynch, 1999), while some others use fixed transaction costs (Morton and Pliska, 1995; Marquering and Verbeek, 2004); or quasi-fixed transaction costs (Duffie and Sun, 1990; Morton and Pliska, 1995).[7] The existence of a wide range of estimates of transaction costs is due to the fact that the true level of transaction costs is difficult to estimate since it depends on many different factors such as: bid-ask spreads; commission rates (different brokerage firms offer different commission fees and internet trading may lowers commission rates); the value and volume of the transaction; the type of investor (institutional vs. individual investors, firm size), share price, etc.

Instead of making several assumptions about transaction costs, we estimate the break-even transaction cost which is less subjective. Essentially, break-even transaction cost, τ^{BE}, is the level of transaction cost that would have just eliminated all the differences in the utilities derived from the dynamic and the static strategies (Han, 2006). The only assumption made here is that the transaction cost is equal to a fixed percentage (τ) of the value traded for all stocks. The break-even transaction cost must be high enough to offset the benefit of dynamic strategy. The higher break-even transaction cost implies the better the dynamic strategy. Therefore, an investor should

[7] Most often, the proportional transaction costs is estimated as half of the sum of both the relative bid-ask spread and commission rate incurred for each transaction. For fixed transaction costs, Marquering and Verbeek (2004) consider three levels of transaction costs, 0.1%, 0.5% and 1%, representing low, medium, and high costs.

only implement the dynamic-timing strategy if he has transaction costs lower than τ^{BE}, otherwise he will be better off with the static strategy. We report the average τ^{BE} in weekly basic points (bps) since it is paid weekly as the portfolio is rebalanced.

3. Data and Descriptive Statistics

Our empirical analysis involves constructing a well-diversified portfolio from weekly returns (Wednesday to Wednesday closing prices) on 34 MSCI stock market indices, over the period from January 13, 1988 to November 28, 2007. Here, we have adopted the perspective of an U.S. investor. The returns are measured in U.S. dollar which means it includes also the exchange rate return between portfolio revisions. We select 34 countries since this is the maximum number of MSCI country equity indices that have the data available for the entire sample period. The countries included in our sample are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Denmark, Finland France, Germany, Hong Kong, Indonesia, Ireland, Italy, Japan, Jordan, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Portugal, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, and United States.

We choose an investment horizon of one week due to the fact we use data from a broad international sample. Working with weekly data alleviates problems associated with non-synchronous trading resulting from time differences between markets located in different geographical regions. All data is obtained from DataStream. We divide the whole sample into two periods: the in-sample estimation period (from January 13, 1988 to December 27, 1995, 416 observations) and the out-of-sample test period (from January 3, 1996 to November 28, 2007, 622 observations). We utilise recursive estimation of the models to generate the one-step ahead forecasts of the covariance. That is the initial estimation from January 13, 1988 to December 27, 1995 produces the forecast for the first observation in the out of sample period, January 3, 1996. We then re-estimate using the sample period January 13, 1988 to January 3, 1996 to produce the next forecast for January 10, 1996 and so on throughout the out-of-sample test period.

Table 1 presents some descriptive statistics of the 34 stock market returns. In general, all the countries have positive average returns. While Japan, New Zealand and Portugal experience the lowest returns (0.11%, 2.19%, 3.97%, respectively), the Latin America countries (Argentina, Brazil, Mexico) that display the highest returns (16.80%, 17.59%, 20.90%, respectively) also happen to be among the most volatile (52.19%, 47.96%, 31.58%). The developed markets have the lowest return volatility (Canada, 16.75%; UK, 16.35%; US, 15.05%) compared with the very high levels of volatility in the emerging markets (e.g. Indonesia, 46.49%, Turkey, 53.81%). All returns (except Japan, Argentina, Indonesia, and Thailand) are negatively skewed, and they all exhibit excess kurtosis. The Jarque-Bera tests clearly reject the null of a Gaussian distribution in all cases. The last five columns report the return sample autocorrelations with lag of 1, 3, 6, 9 and 12. None of the autocorrelation coefficients is statistically significant.

4. Empirical Results

4.1 Jumps estimation

We begin our empirical analyses by estimating the stochastic volatility with correlated jumps (SVCJ) model in equations (3) and (4) for 34 countries over the full sample period from January 13, 1988 to November 28, 2007. The model is estimated using the efficient Markov Chain Monte Carlo (MCMC) procedure proposed by Raggi

(2005), which implements the Delay Rejection algorithm for 250,000 iterations and burn-in the first 50,000 to completely remove the effect of initial values.

Table 2 reports the mean posterior parameter estimates for the SVCJ model in equations (3) and (4). There are several interesting features of the parameter estimates. Overall, there is a wide dispersion in parameter estimates among the 34 countries. The long-term means of volatility process (θ) in developed markets are very much lower than in the emerging markets. For example, the long-term volatility estimates are low for the US (13.06%), Canada (15.12%), the Netherlands (15.20) and the UK (17.85%), but are very high for Korea (29.69%), Brazil (32.26%), Argentina (35.41%) and Turkey (68.05%). There are also large differences among countries in terms of the intensity of jump process (λ) and the number of jumps detected.[8] Specifically, the model expects only around 0.76 and 0.91 jumps per year for the Netherlands and the UK while the expected number of jumps rises to 8, 9, and 9 jumps per year for Thailand, Argentina and Brazil, respectively. As expected, the model detects a large number of jumps in countries that have experienced currency or financial crises such as Indonesia (53 jumps), Korea (70), Mexico (51), the Philippines (67), and Thailand (64). On the other hand, and also in line with expectations, the least number of jumps detected are in more developed and stable markets such as Australia (12 jumps), Denmark (12), and the UK (10). In general, our relatively high estimates of volatilities and jumps for most of the countries are reasonable given our sample period lies in the very volatile markets due to the Mexican currency crisis and its Tequila effect (1995-1996), the South East Asian

[8] Following Maheu and McCurdy (2004), we use probability value 0.5 as the threshold level for detecting jumps. That is, a jump occurs as the estimated probability of jump equal to or greater than 0.5. This probability is derived from $1 - P_c(J_t = 1)$ in equation (A.14) in Appendix A.

financial crisis (1997-1998), the terrorist attack in the US (11, September 2002) and the bear markets in 2002.

Regarding the jump parameters, we observe that the jump size correlation (ρ_j) between return jump size and volatility jump size are mostly positive which means that an increase in the jump size of volatility could induce an increase in the jump size of return.[9] In addition, the results also indicate that mean jump sizes of return H_t^y given by $\phi_y + \rho_j H_t^v$ are higher than mean jump size of volatility H_t^v measured by $\exp(\phi_v)$ in most of the countries, and they are often much higher in the highly volatile markets such as Argentina, Korea and Indonesia, which is understandable. Average jump size for both returns and volatility is much smaller in magnitude for developed markets such as Germany, the US and UK.

One interesting question is how frequently do market jump together, that is co-jump? Figure 1 provides some insight. It reports the number of markets jumping during each week of the sample period. For the majority of the sample 571 weeks (55%) there are no jumps observed, this is followed by a further 241 weeks (23%) where only 1 market experiences an isolated jump. However there are 52 weeks (5%) where 5 or more markets all experience a jump. Figure 1 also identifies events where 10 or more markets experience co-jumps, these coincide with many documented "crises" or significant systemic events of the past two decades.

4.2 Economic value of volatility and jump timing

To appreciate the economic value of controlling for jumps in asset allocations we need to compare the performance of the dynamic volatility-jump-timing strategy with other strategies that do not account for jumps (static and dynamic volatility-timing strategies). After all the jumps have been detected and removed in the previous section, we employ the asymmetric generalised dynamic conditional correlation GARCH model (AG-DCC-GARCH) of Capiello et al. (2006) to estimate and forecast the conditional variance-covariance matrices for the original return series as well as the jump-removed return series.[10,11] Next, to implement the static and dynamic asset allocation strategies, we need a vector of expected returns. However, returns are uncertain and difficult to predict. As Fleming, Kirby and Ostdiek (2001) show a single estimate of this vector is unlikely to be accurate, we therefore follow their recommendation and use unconditional expected returns that are generated via a bootstrap procedure. Specifically:

Step 1: Create a sample of n = 4,000 returns for each stock market by randomly picking up blocks of 10 observations from the in sample period of the return series, then estimate the unconditional mean return vector and unconditional covariance matrix for the 34 markets.

Step 2: The estimated unconditional mean return vector and unconditional covariance matrix are used to compute the optimal portfolio weights for the static strategy. The unconditional mean return vector and conditional covariance matrix (via AG-DCC-GARCH) are used to calculate the optimal portfolio weights for the dynamic-timing

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[9] Following Eraker, Johannes and Polson (2003), the prior distribution of ρ_j is set equal to $N(0,4)$.

[10] To save space we do not report the estimates of variances and covariances. The results are available upon request.

[11] Only the in-sample estimation period has jumps removed. Unadjusted returns are used in the out-of-sample evaluation period.

strategies under the maximum-return objective and the minimum-volatility objective.

Step 3: Apply the optimal portfolio weights above to the raw returns to calculate portfolio realized returns in the next period and the various performance measures (Sharpe ratios, performance fees, and break-even transaction costs) for each strategy.

Step 4: Repeat step 1 through step 3 1000 times, saving each of the performance measures estimates. Report the mean value for each of the performance measures.

Since all the strategies are subject to the same amount of estimation risk from the bootstrap procedure, their relative performance reflects the gains attributable to volatility-timing or volatility-jump timing. Figure 2 plots the Sharpe ratios from the 1,000 simulation trials (each with n = 4,000 observations) for the maximum-return objective with a target volatility of 10% and for the minimum-volatility objective with a target return of 12%. Each point in Figure 2 represents a separate trial, which is the realized Sharpe ratio for both the static (horizontal axis) and dynamic (vertical axis). All four figures show convincingly that the Sharpe ratios are clustered above the 45-degree line, indicating that the dynamic strategies strongly outperform the static ones in the vast majority of the 1,000 trials. Furthermore, these plots clearly suggest that the volatility-jump-timing strategy (on the right) outperforms the volatility-timing strategy that does not account for jumps. For instance, under the maximum return objective, 93.05% of the Sharpe ratios from the volatility-jump-timing strategy are higher than that of the static strategy, while only 88.37% are higher for the volatility-timing strategy.

This difference is greater for the minimum volatility objective. Here, 94.98% from the volatility-jump-timing strategy are higher compared with 86.72% from the volatility-timing strategy.

Next, we analyse and compare the detailed performance of the different portfolio allocation strategies using the three sets of performance measures. Table 3 shows the out-of-sample performance comparison of the volatility-timing, volatility-jump-timing and the static strategies under the maximum return strategy for a range of target volatilities from 8%-16%. Similar to Fleming, Kirby and Ostdiek (2001), we find volatility-timing outperform the static portfolios for most levels of target volatilities. In virtually all cases, the volatility-jump-timing strategy outperforms both volatility-timing and static strategies.

Take for instance, the case where the target volatility is 10% per year. At this volatility level, the Sharpe ratio of volatility-jump-timing strategy is 0.78, which is higher than the Sharpe ratio of volatility-timing strategy (0.73) and static strategy (0.64). Moreover, a small improvement in Sharpe ratios of volatility-jump-timing strategy compared to the volatility-timing strategy can translate to a sizable improvement in both performance fees (Δ_λ) and break-even transaction costs (τ_λ^{BE}). For the case of γ=1, for example, the strategy accounting for jumps achieves an improvement in Sharpe ratios of 0.05 (=0.78-0.73), which translates into 26.8 (=194.5-167.7) annual basis points (bps) of performance fees and a 0.79 (=8.93-8.14) weekly bps increase in break-even transaction costs. That implies the investor would be willing to pay up to an extra of 26.8 annual bps of performance fees or up to an extra of 0.79 weekly bps of break-even transaction costs to switch from a volatility-timing strategy to volatility-jump-timing strategy.

Interestingly, we observe that as the target volatility level increases, the performance fees ($\Delta_{\gamma=1}$) and break-even transaction costs ($\tau_{\gamma=1}{}^{BE}$) for our low risk aversion investors (γ=1) increase while the reverse is true for the high risk aversion investors (γ=10). These results reflect the trade-off between risk and return, and are consistent with the findings in Fleming, Kirby and Ostdiek (2000). In addition, we can see that the investor with high level of relative risk aversion (γ=10) prefers the static model when volatility level is very high. This type of investor would reject the volatility-timing strategy when the target volatility level increase to 15%, and would reject the volatility-jump-timing when the target volatility level reaches 16%.

For the minimum-volatility objective (Table 4), the performance results also support asset allocation strategy that controls for jumps. Averaging across the target return levels, the improvements in Sharpe ratios of the dynamic-timing strategies are comparable (or slightly higher) with that under the maximum return strategies. However, the performance fees and especially the break-even transaction costs estimates are, on average, less than that found under the maximum return strategies. For example, at 12% level of target return, in terms of break-even transaction costs, our quadratic-utility investor with the low risk aversion of 1 would be willing to pay extra of only 0.80 or 0.90 weekly bps, respectively, to switch to a volatility-timing strategy or a volatility-jump-timing strategy.

As a whole, our analysis of performance measures under different objectives shows that accounting for jumps in portfolio allocations has sizeable economic value. It would also be interesting to see how the strategy that controls for jumps performs over different periods of time and market conditions. Table 5 presents the year-by-year comparison of the performance of the two dynamic strategies with the static strategy over the period from 1996 to 2007. The table reports the results for the maximum return strategy with a targeted volatility of 12%. Overall, it is clear that the performances of the dynamic strategies are much stronger than the static one in all the periods. Comparing the dynamic-timing strategies, it is also clear that the volatility-jump-timing strategy outperforms its counterparties in all the years except 1998, when the numbers of jumps detected (by the SVCJ model) reaches the highest level (141 jumps for all the countries). More specifically, in 1998, although the Sharpe ratios are similar (-0.1%) across strategies, the performance fees (Δ_1) and the break-even transaction costs (τ_1^{BE}) of volatility-timing strategy are higher than that of volatility-jump-timing strategy (207.7 *cf.* 195.4 annual bps, and 10.14 *cf.* 8.22 weekly bps, respectively for γ=1). The results from minimum-volatility objective, details not reported here o conserve space, also show similar results.

All in all, our results suggest that controlling for jumps in portfolio allocations provides significant economic value. Only in extremely volatile periods when there are large numbers of simultaneous jumps is the volatility-jump-timing strategy outperformed by the volatility-timing strategy.[12] Indeed, our descriptive statistics by year (not shown) show that the sample volatility is extremely high in 1998, reaching 36% (highest). Also in this year, there are 7 international systemic jumps (the highest number of systemic jumps in all the period examined).[13] During the Asian crisis, jump was very persistence in a handful of Asian countries. Removing jumps has distorted the volatility estimates and the correlation relationships of these countries vis-a-vis

[12] In our case the maximum number of jumps which still makes volatility-jump-timing strategy preferable compared with the volatility-timing strategy lies between 100-120 jumps per year.

[13] We define market or systemic jumps as occasions when there are simultaneous jumps in more than 5 stock markets in the same week.

countries not affected by the crisis, and led to less superior portfolio performance. It is worth noting that both dynamic strategies still outperform the static strategy.

To illustrate the impact of the Asian crisis, we show the comparison of volatility forecasts and portfolio weight estimates for the two competing dynamic-timing strategies (one adjusted for jump (AJ) and the other does not). Out of the 34 countries included in the portfolio, we find that, during the 1997-1998 Asian financial crisis, the jump-adjusted strategy severely underestimates the volatility of the five crisis-affected countries: Hong Kong, Indonesia, Korea, Malaysia and the Philippines (see the left side of Figure 3). This problem occurs throughout the year 1998 for the five countries. As a result, the jump-adjusted strategy tends to maintain higher levels of portfolio weight for these countries during the year 1998 compared to the competing strategy (see the right side of Figure 3). Indeed, Figure 3 shows not only that the non-jump-adjusted strategy has lowers portfolio weights for the five countries; it has more short positions in these stocks for parts of 1998, particularly for Indonesia, Korea, Malaysia and the Philippines.

The underestimation of volatility during the crisis period gives rise to the underperformance of the jump-adjusted strategy. For example, during the crisis period (from July 1997 to December 1998), these five countries, viz. Hong Kong, Indonesia, Korea, Malaysia and the Philippines, have, respectively, annualized returns of -43.62 %, -182.35 %, -34.18 %, -142.98 %, -75.02 %. These five countries alone account for a total loss of -9.15% in difference in portfolio annualized return between volatility-jump strategy and the volatility timing strategy.

4.3 Robustness Analysis

In this section we present a robustness check of our results. Thus far, all reported results are based on jumps detected using the stochastic volatility with correlated jumps

(SVCJ) model (Duffie et al., 2000) estimated using the efficient Markov Chain Monte Carlo (MCMC) procedure proposed by Raggi (2005). One could argue that different methods of estimating jumps might lead to different jump estimation and hence if there is a problem with our model and approach (such as keeping the jump intensity constant over time) our asset allocation results could be unrealizable.

To investigate this issue, we use a different approach to estimate jumps to examine whether our results are robust to jump assumption and estimation. We use a parametric mixed-GARCH-Jump-Intensity (GARJI) model proposed by Chan and Maheu (2002) and Maheu and McCurdy (2004) which overcomes the constant jump intensity problem in our SVCJ model. Indeed, the GARJI model allows the expected arrival rate of jumps to cluster and to vary over time. Moreover, this model allows for asymmetric responses to past return innovations in the GARCH components of volatility. Finally, the variance equation of this model includes the effect of both past normal innovations and past jump innovations to returns (details on the model and estimation procedures are provided in the Appendix).

We estimate the GARJI model for 34 country returns over the full sample period. Overall, we find that this model detects about 6% more jumps than that detected by the SVCJ model. We then re-estimate the conditional variance-covariance matrices after removing the detected jumps as before. Finally, we re-estimate the performance measures based on the new variance-covariance matrices. Tables 6 and 7 present the out-of-sample performance comparison of the volatility-timing, the volatility-jump-timing and the static strategies under the maximum-return objective and the minimum-volatility objective, respectively. Generally speaking, there is not much difference between the results obtained using SVCJ model and those obtained employing the

GARJI model. It can be seen, again, that the volatility-jump-timing strategy outperforms volatility-timing at all levels of target volatilities (maximum-return objective) as well as all levels of target returns (minimum-volatility objective). Compared with the results obtained by the SVCJ model, there are slight improvements in the performance measure estimates using the GARJI model.

These new findings suggest that our previous results are robust to different jump estimation methods. Provided that jumps are correctly identified, control for jumps in international portfolio allocations could bring about substantial economic value.

5. Conclusion

In the last few years, two strands of literature have been developed in parallel; one on the discovery of jumps in stock returns, and more recently jumps in volatility. The other strand of research focuses on various ways of modelling conditional variance and higher moments in the hope of leading to a better portfolio choice. Here, we argue that jumps, like outliers, are rare, extreme and unpredictable. If not identified and removed, jumps can severely bias variance and covariance estimates. Jumps are also responsible for unstable higher moments.

This paper empirically investigates the presence of jumps in international equity returns and volatility. First we clearly document the prevalence of jumps in international stock markets and also show the extent to which co-jumping occurs. We then examine if controlling for jumps can lead to a better portfolio choice. We test our hypothesis by constructing portfolio out of 34 international stock market return series, and measuring performance based on the Sharpe ratio, the utility based performance fee and a break-even transaction cost. We compare three strategies, viz. static, dynamic and dynamic-jump-removed. In the third strategy, we remove jumps before estimating the conditional covariance matrix. We estimate jumps using a robust Markov Chain Monte Carlo method as well as a Mixed-GARCH-Jump-Intensity model. All the portfolio evaluations are based on actual outcome (i.e. without removing jumps from portfolio actual returns). The results show consistently that controlling for jumps leads to better estimates of the variance-covariance matrix and better portfolio performance. The only exception to this rule is when the markets are extremely volatile and the portfolio performance is overshadowed by jump persistence. In 1998, the jump removed estimation procedure always gives a lower variance forecast for a handful of countries involved in the Asian crisis. The premise of our paper is based on jumps being unpredictable. One way to extend our work is to develop models or portfolio strategies that will pick up jump persistence. Further, given events of the recent Global Financial Crisis it would be interesting to examine the performance of the volatility-jump strategy on an extended sample.

One may argue that the jump estimation result would be very different should we use daily or monthly returns instead of weekly return. It is likely that as data frequency gets lower, the jumps will get “averaged out” and at the same time the power of the test to detect reduces due to the smaller number of observations. We believe a weekly horizon is appropriate. In practice, the choice of data frequency should be determined by the portfolio revision period.

Appendix A

MCMC Procedure for Stochastic Volatility Correlation Jump Model (SVCJ)

The SVCI model specification and estimation procedures follow those in Duffie, Pan and Singleton (2000) and Raggi (2005).

A.1 The SVCJ model:

$$dY_t = \left(\phi - \frac{1}{2}V_t\right)dt + \sqrt{V_t}dW_t^y + dJ_t^y \tag{A.1}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v \sqrt{V_t}dW_t^v + dJ_t^v \tag{A.2}$$

$$dJ_t^i = H_t^i dN_t^i, \qquad i = \{y, v\} \tag{A.3}$$

where Y_t is equity log-prices, V_t is volatility process; W_t^y and W_t^v are two correlated Brownian motions with correlation $\rho = E[W_t^y W_t^v]$; $dJ_t^i = H_t^i dN_t^i$, $i = \{y, v\}$ is the jump term with a jump-size component H_t^i, and N_t^i is the number of arrivals (of a Poisson process in the interval $(0, t]$). All the other variables, ϕ, κ, θ and σ_v, are parameter constants.

In this paper, jumps in volatility and returns are driven by the same Poisson process ($N_t^y = N_t^v$) with constant jump intensities $\lambda_v = \lambda_y$ and jump size distributions:

$$H_t^v \sim \exp(\phi_v), \tag{A.4}$$

$$H_t^y \mid H_t^v \sim N(\phi_y + \rho_j H_t^v, \sigma_y^2). \tag{A.5}$$

The jump sizes, H_t^y and H_t^v, are different but correlated with coefficient ρ_j.

To avoid problems with negative volatilities, we consider the logarithmic transform $\log(V_t) = Z_t$ with

$$d\,\mathrm{Z_t} = \left[\kappa(\theta e^{-Z_t} - 1) - \frac{1}{2}\sigma_v^2 e^{-Z_t} \right] dt + \sigma_v e^{-\frac{1}{2}Z_t} dW_{2,t} + \log(1 + H_t^v e^{Z_{t-}}) dN_t^v, \qquad \text{(A.6)}$$

where $e^{Z_{t-}}$ is the left limit of a trajectory.

A.2 Inference

The first step of the inference procedure involves the transformation of the continuous differential equations into discrete difference equations (Euler discretization). The model is then estimated on a set of discrete times $\{\tau_i : i = 1, \cdots, N\}$ such that $\tau_{i+1} - \tau_i = \Delta$. In this paper, we use weekly intervals with $\Delta = \tau_{i+1} - \tau_i = (t+1) - t = 1$, and

$$Y_{t+1} - Y_t = \left(\phi - \frac{1}{2} V_t \right) + \sqrt{V_t} \varepsilon_{t+1}^y + H_{t+1}^y J_{t+1}^y \qquad \text{(A.7)}$$

$$V_{t+1} - V_t = \kappa(\theta - V_t) + \sigma_v \sqrt{V_t} \varepsilon_{t+1}^v + H_{t+1}^v J_{t+1}^v ., \qquad \text{(A.8)}$$

where $(\varepsilon_{t+1}^y, \varepsilon_{t+1}^v)$ is a bivariate Normal with correlation ρ, and J_{t+1} is Binomial $Bi(1, \lambda)$.

Since we use log-volatilities in equation (A.6), more transformation is needed (Glasserman and Merener, 2004):

$$Y_{t+1} = \phi - \frac{1}{2} e^{Z_t} + e^{\frac{1}{2}Z_t} \varepsilon_{t+1}^y + H_{t+1}^y J_{t+1}^y, \qquad \text{(A.9)}$$

$$\mathrm{Z_{t+1^-}} = \mathrm{Z_t} + \kappa(\theta e^{-Z_t} - 1) - \frac{1}{2}\sigma_v^2 e^{-Z_t} + \sigma_v e^{-\frac{1}{2}Z_t} \varepsilon_{t+1}^v, \qquad \text{(A.10)}$$

$$\mathrm{Z_{t+1^-}} = \mathrm{Z_{t+1^-}} + \log(1 + H_t^v e^{Z_{t+1-}}) J_{t+1}^v \quad . \qquad \text{(A.11)}$$

In this paper, equations (A.9)-(A.11) are used to evaluate the likelihood function through auxiliary particle filter. Using the sequential Monte Carlo simulations, particle filter estimates the hidden volatility and jump parameters based on observation $Y = \{Y_1, ..., Y_T\}$. Similar definitions apply to the vector of stochastic volatilities V and the jump process J^i with jump sizes H^i. In the estimation, volatilities and jumps are considered as missing data with the target distribution $\psi(\theta, V, H^v, H^y \mid Y)$.

MCMC aims at estimating the distribution $p(\theta \mid Y)$ by recursively simulating the latent processes and their parameters from their full conditional distributions. The latent factors are estimated by averaging over the realization of chains:

$$\hat{V}_t = n^{-1} \sum_{j=1}^{n} V_t^j, \tag{A.12}$$

$$\hat{\Pr}(J_t = 1) = n^{-1} \sum_{j=1}^{n} J_t^j. \tag{A.13}$$

The initiating values are set as[14] $\phi \sim N(0,25)$, $\kappa \sim N(0,1)$, $\kappa\theta \sim N(0,1)$, $\sigma_v^2 \sim U(0,1)$, $\rho \sim U(-1,1)$, $\lambda \sim Beta(2,40)$, $\rho_j \sim N(0,4)$, $\phi_v \sim IG(20,10)$, $\sigma_y^2 \sim IG(5,20)$, $\phi_y \sim N(0,100)$. An adaptive Metropolis-Hasting algorithm (ARMS) was used to update the parameters. The Delay Rejection algorithm proposed by Tierney and Mira (1999) was used to increase the rate of acceptance at each sweep of the algorithm. For example, in the case of a rejection, the idea is to sample a new candidate that takes into account the information given by the previous rejections.

Finally, because the jump times J_t are a sequence of i.i.d Bernoulli random variables, the conditional posterior is Bernoulli with the probability of success given by:

$$P_c(J_t = 1) = \frac{P(J_t = 1 \mid \Phi, J_{-t})}{P(J_t = 0 \mid \Phi, J_{-t}) + P(J_t = 1 \mid \Phi, J_{-t})}, \tag{A.14}$$

where $\Phi = \{\theta, Y, V, H^v, H^y\}$ and $P(J_t = j \mid \Phi, J_{-t})$ for $j = 0,1$ is known up to a proportionality term. J_{-t} comprises all observations on the jump process J except J_t, i.e. $J_{-t} = \{J_1, \ldots, J_{t-1}, J_{t+1}, \ldots, J_T\}$.

[14] The symbol IG(.) indicates an Inverse Gamma random variable.

Appendix B

Mixed-GARCH-Jump Model

In this paper, the modelling framework and estimation procedures of the Mixed-Jump-Intensity model follow those in Chan and Maheu (2002) and Maheu and McCurdy (2004).

B.1 Return process:

The Mixed-GARCH-Jump-Intensity (or GARJI) model combines a jump specification with a GARCH parameterization of the volatility. The return process is assumed to be:

$$r_t = \mu + \sum_{i=1}^{L} \varphi r_{t-i} + \varepsilon_{1,t} + \varepsilon_{2,t}, \qquad \text{(B.1)}$$

where $\varepsilon_{1,t}$ is a "normal" innovation which drives smooth changes in returns, i.e. it is a mean zero innovation with a normal stochastic process, $\varepsilon_{2,t}$ is a "surprise" innovation which causes infrequent large changes (jumps) in returns.

The "normal" innovation has a GARCH (1,1) structure with conditional variance h_t:

$$\varepsilon_{1,t} = \sqrt{h_t}\, z_t, \quad z_t \sim iid\, N(0,1)$$

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i\, h_{t-i}, \qquad \text{(B.2)}$$

where $\varepsilon_{t-1} = \varepsilon_{1,t-1} + \varepsilon_{2,t-1} = r_{t-1} - \mu - \sum_{i=2}^{L} \varphi r_{t-i}$.

The "surprise" innovation follows a Poison jump process,

$$\varepsilon_{2,t} = \sum_{k=1}^{n_t} Y_{t,k},$$

where $Y_{t,k}$ denotes the conditional jump size with $Y_{t,k} \sim iid\ N(\theta, \delta_t^2)$, n_t, the number of jumps that arrive between t-1 and t, has probability density:

$$P(n_t = j \mid \Phi_{t-1}) = \frac{\varepsilon^{-\lambda_t} \lambda_t^j}{j!}, \qquad j = 1, 2... \tag{B.3}$$

where $\lambda_t \equiv E[n_t \mid \Phi_{t-1}] \geq 0$, the jump intensity, is assumed to be time-varying and follows an Autoregressive Conditional Jump Intensity process, ARJI(1,1):

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\zeta. \tag{B.4}$$

The jump intensity residual ζ_{t-1}, in turns, is defined as:

$$\begin{aligned} \zeta_{t-1} &= E[n_t \mid \Phi_{t-1}] - E[n_t \mid \Phi_{t-2}] \\ &= \sum_{j=0}^{\infty} jP(n_{t-1} = j \mid \Phi_{t-1}) - \lambda_{t-1} \end{aligned} \tag{B.5}$$

where $P(n_{t-1} = j \mid \Phi_{t-1})$ is a filter specified in (B.12). Thus the conditional mean of the jumps component is:

$$E(\varepsilon_{2,t}) = \theta\lambda \quad , \tag{B.6}$$

while the conditional variance of the jumps component is:

$$Var(\varepsilon_{2,t} \mid \Phi_{t-1}) = \delta_t^2 \lambda_t \quad . \tag{B.7}$$

B.2 Conditional mean and variance of returns (used for the Likelihood function)

The conditional mean of the returns process can be derived from equations (B.1) and (B.6) as

$$E(r_t \mid \Phi_{t-1}) = \mu + \sum_{i=1}^{L} \varphi r_{t-i} + \theta\lambda_t \quad . \tag{B.8}$$

Consequently, the conditional variance of returns is calculated as:

$$Var(r_t \mid \Phi_{t-1}) = Var(\varepsilon_{1,t} \mid \Phi_{t-1}) + Var(\varepsilon_{2,t} \mid \Phi_{t-1}) \tag{B.9}$$

The variance equation of the GARJI model includes the effect of both past normal innovations and past jump innovations to returns. In other words, the GARJI model estimates the number of jumps that occurred during period *t*-1 to *t* and allow it to directly affect the feedback that ε_{t-1} has on the GARCH variance in (B.9).

B.3 Likelihood function

The distribution of returns, conditional on the most recent information and *j* jumps, is normally distributed:

$$f(r_t \mid n_t = j, \Phi_{t-1}) = \frac{1}{\sqrt{2\pi(h_t + j\delta_t^2)}} \exp\left(-\frac{(r_t - \mu - \sum_{i=1}^{L} \varphi r_{t-i} - \theta_t j)^2}{2(h_t + j\delta_t^2)} \right) \tag{B.10}$$

Thus, the conditional density of returns, unconditional on jumps, is:

$$f(r_t \mid \Phi_{t-1}) = \sum_{j=0}^{\infty} f(r_t \mid n_t = j, \Phi_{t-1}) P(n_t = j \mid \Phi_{t-1}) \tag{B.11}$$

where $P(n_t = j \mid \Phi_{t-1})$ is from (B.2). Given the observed r_t and using Bayes rule, we can derive the ex post probability of the occurrence of *j* jumps at time *t* with the filter:

$$P(n_t = j \mid \Phi_t) = \frac{f(r_t \mid n_t, \Phi_{t-1}) P(n_t = j \mid \Phi_{t-1})}{f(r_t \mid \Phi_{t-1})} \quad . \tag{B.12}$$

References

Ang, A., Bekaert, G., 2002. International asset allocation with regime shifts. *Review of Financial Studies* 15, 1137-1187.

Balduzzi, P., Lynch, A.W., 1999. Transaction costs and predictability: Some utility cost calculations. *Journal of Financial Economics*, 52, 47–78.

Bates, D., 2000. Post-'87 crash fears in S&P500 futures options. *Journal of Econometrics* 94, 181-238.

Bhardwaj, R., Brooks, L., 1992. The January anomaly: effects of low share price, transaction costs and bid-ask bias. *Journal of Finance* 47, 553–574.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 31, 307–328.

Broadie, M., Chernov, M., Johannes, M., 2007. Model specification and risk premia: Evidence from futures options. *Journal of Finance* 62, 1453–1490.

Cappiello, L., Engle, R.F., Sheppard, K., 2006. Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics* 4, 537-572.

Chan, W.H., Maheu, J.M., 2002. Conditional jump dynamics in stock market returns. *Journal of Business and Economic Statistics* 20, 377-89.

Constantinides, G., 1986. Capital market equilibrium with transaction costs. *Journal of Political Economy* 94, 842–862.

Das, S.R., Uppal, R., 2004. Systemic risk and international portfolio choice. *Journal of Finance* 59, 2809-2834.

De Goeij, P., Marquering, W., 2004. Modeling the conditional covariance between stock and bond returns: A multivariate GARCH approach. *Journal of Financial Econometrics* 2, 531–564.

Della Corte, P., Sarno, L., Tsiakias, I., 2008. An economic evaluation of empirical exchange rate models: Robust evidence of return predictability and volatility timing. *Review of Financial Studies*, forthcoming.

Duffie, D., Pan, J., Singleton, K., 2000. Trasform analysis and asset pricing for affine jump-diffusions. *Econometrica* 68, 1343–76.

Duffie, D., Sun, T., 1990. Transaction costs and portfolio choice in a discrete continuous-time setting. *Journal of Economic Dynamics and Control* 14, 35–51.

Engle, Robert F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* 50, 987–1008.

Engle, R.F., 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive-conditional heteroskedasticity models. *Journal of Business and Economic Statistics* 20, 339-350.

Eraker, B., 2004. Do equity prices and volatility jumps? Reconciling evidence from spot and option prices. *Journal of Finance* 59, 1367–403.

Eraker, B., M. Johannes and N. Polson, 2003. The impact of jumps in volatility and returns. *Journal of Finance* 58, 1269–300.

Fama, E.F., and MacBeth, J.D, 1973. Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy* 81, 607–636.

Fleming, J., Kirby, C., Ostdiek, B., 2001. The economic value of volatility timing. *Journal of Finance* 56, 329-352.

Fleming, J., Kirby, C., Ostdiek, B., 2003. The economic value of volatility timing using "realized" volatility. *Journal of Financial Economics* 67, 473-509.

Forster, D.P., Nelson, D.B., 1996. Continuous record asymptotics for rolling sample variance estimators. *Econometrica* 64, 139-174.

Glasserman, P. & N. Merener, 2004. Convergence of a discretization scheme for jump-diffusion processes with state-dependent intensities, In Proceedings of the Royal Society, Vol 460, 111–127.

Guidolin, M., Timmermann, A., 2008. International asset allocation under regime switching, skew and kurtosis preferences. *Review of Financial Studies*, forthcoming.

Han, Y., 2006. Asset Allocation with a high dimensional latent factor stochastic volatility model. *Review of Financial Studies* 19, 237–271.

Johannes, M. Polson, N., Stroud, J., 2002. Sequential optimal portfolio performance: Market and volatility timing. Working paper. Columbia University.

Jondeau, E., Rockinger, M., 2007. The economic value of distributional timing. Working paper. HEC Lausanne.

Jorion, P., 1988. On jump processes in the foreign exchange and stock markets. *Review of Financial Studies* 1, 427-445.

Maheu, J.M., McCurdy, T.H., 2004. News arrival, jump dynamics, and volatility components for individual stock returns. *Journal of Finance* 59, 755-793.

Marquering, W., Verbeek, M., 2004. The economic value of predicting stock returns and volatility. *Journal of Financial and Quantitative Analysis* 39, 407-429.

Morton, A.J., Pliska, S.R., 1995. Optimal portfolio management with fixed transaction costs. *Mathematical Finance* 5, 337-356.

Officer, R.R., 1973. The variability of the market factor of the New York Stock Exchange. *Journal of Business* 46, 434–453.

Pan, J., 2002. The jump-risk premia implicit in options: Evidence from an integrated time-series study. *Journal of Financial Economics* 3, 125-144.

Raggi, D., 2005. Adaptive MCMC methods for inference on affine stochastic volatility models with jumps. *Econometrics Journal* 8, 235–250.

Stoll, H., Whaley, R., 1983. Transaction costs and the small firm effect. *Journal of Financial Economics* 12, 57–79.

Tierney, L., and Mira, A., 1999. Some adaptive Monte Carlo methods for Bayesian inference, *Statistics in Medicine* 8, 2507-2515.

West, K.D., Edison, H.J., Cho, D., 1993. A utility-based comparison of some models of exchange rate volatility. *Journal of International Economics* 35, 23-45.

Zakoian, J., 1994. Threshold heteroskedastic models, *Journal of Economic Dynamics and Control* 18, 931-55.

Wu, L., 2003. Jumps and dynamic asset allocation, *Review of Quantitative Finance and Accounting* 20, 207-243.

Table 1. Summary statistics for weekly MSCI stock index returns

from the period 13 January, 1988 to 28 November, 2007

Country	Mean	Std Dev	Skew	Kurt	Jarque-Bera	*ρ1*	*ρ3*	*ρ6*	*ρ9*	*ρ12*
Argentina	16.8	52.19	1.08	17.76	13644*	-0.113	0.048	-0.102	-0.053	-0.007
Australia	9.11	17.79	-0.34	0.9	54*	-0.02	0.007	-0.055	0.013	0.009
Austria	9.58	21.43	-0.32	2.73	332*	0.022	0.003	-0.048	-0.015	-0.065
Belgium	8.56	19.24	-0.43	4.1	743*	-0.116	0.021	-0.022	0.028	-0.045
Brazil	17.59	47.96	-1.14	6.76	2165*	0.081	-0.052	-0.033	-0.036	-0.037
Canada	9.15	16.75	-0.39	1.62	137*	0.042	0.032	0.02	0.039	0.012
Chile	13.96	23.08	-0.1	2.09	185*	0.08	0.062	-0.012	-0.011	-0.042
Denmark	13.42	18.62	-0.56	2.35	288*	-0.089	0.015	-0.025	0.008	-0.072
Finland	11.89	32.59	-0.52	3.87	683*	-0.035	0.053	0.041	-0.015	-0.007
France	10.2	19.86	-0.35	2.31	246*	-0.136	0.028	-0.023	0.058	-0.085
Germany	9.8	21.36	-0.49	2.36	277*	-0.092	0.016	-0.002	0.017	-0.065
Hong Kong	10.14	24.26	-0.84	3.87	757*	0.031	0.055	-0.034	0.011	-0.058
Indonesia	9.5	46.49	1.23	28.99	36100*	0.032	0.094	0.03	-0.02	-0.005
Ireland	7.36	21	-0.34	1.33	94*	-0.071	0.034	-0.034	-0.018	-0.032
Italy	6.04	22.55	-0.37	2.19	226*	-0.059	0.075	-0.033	0.006	-0.038
Japan	0.11	21.72	0.17	1.22	68*	-0.011	0.02	-0.038	0.018	-0.02
Jordan	4.54	17.36	-2	28.64	35660*	0.106	0.045	0.028	0.018	0.028
Korea	7.16	34.47	-0.56	6.58	1898*	-0.031	0.095	-0.002	-0.097	-0.051
Malaysia	6.37	29.77	-0.77	15.46	10293*	0.049	0.008	0.034	0.046	-0.039
Mexico	20.9	31.58	-0.2	2.93	370*	0.049	0.083	0.002	-0.002	-0.083
Netherlands	9.37	18.32	-0.71	5.53	1387*	-0.169	-0.013	0.022	0.036	-0.055
New Zealand	2.19	21.21	-0.19	1.88	156*	0.013	-0.014	-0.026	0.006	0.018
Norway	10.95	22.84	-0.46	1.78	169*	0.012	-0.01	-0.043	0.058	-0.011
Philippines	6.02	30.56	-0.05	3.59	548*	0.026	0.101	0.025	0.008	-0.043
Portugal	3.97	20.04	-0.04	1.43	86*	0.034	0.006	-0.023	0.02	0.002
Singapore	8.19	21.35	-0.35	2.37	258*	0.067	0.005	0.012	-0.058	-0.017
Spain	9.36	21.21	-0.42	1.67	149*	-0.063	0.009	-0.036	0.034	-0.069
Sweden	12.11	25.52	-0.51	2.88	394*	-0.093	0.06	0.05	-0.02	0.021
Switzerland	11.21	17.5	-0.38	2.07	205*	-0.076	0.019	0.013	0.052	-0.05
Taiwan	5.14	34.19	-0.32	2.28	238*	0.062	0.007	-0.016	0.011	0.054
Thailand	4.08	35.71	0.01	2.27	218*	0.039	0.077	0.044	-0.015	-0.038
Turkey	9.82	53.81	-0.19	1.66	123*	0.067	0.065	-0.026	-0.011	-0.022
United Kingdom	6.94	16.35	-0.02	1.99	167*	-0.108	0.036	0.002	0.013	-0.035
United States	8.79	15.02	-0.23	2.11	197*	-0.109	0.023	0.03	0.038	-0.019

Note: The means and standard deviations (Std Dev) are annualized and reported in percentage. 'Skew' is skewness, 'Kurt' is Kurtosis, *ρi* is the lag-*i* autocorrelation of change in conditional volatility. '*' indicates significance at the 1 percent level.

Table 2. Parameter Estimates of the SVCJ Model for weekly MSCI stock index returns from the period 13 January, 1988 to 28 November, 2007

$$dY_t = \left(\phi - \frac{1}{2}V_t\right)dt + \sqrt{V_t}\,dW_t^y + dJ_t^y$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v\sqrt{V_t}\,dW_t^v + dJ_t^v$$

Country	ϕ	κ	θ	σ_v	λ	ρ	ρ_j	ϕ_v	ϕ_y	σ_y	Jumps detected
Argentina	18.21	0.29	35.41	7.07	8.99	0.12	1.63	3.32	-0.15	9.96	41
Australia	12.18	0.14	20.64	5.37	2.15	0.12	0.24	0.73	5.30	2.64	12
Austria	11.64	0.25	20.95	5.43	2.31	0.00	-0.45	0.69	0.64	2.31	25
Belgium	10.72	0.18	15.12	5.41	2.09	0.49	1.44	0.63	3.63	2.91	29
Brazil	19.63	0.27	32.26	7.08	9.27	0.07	-0.20	2.84	2.24	10.20	32
Canada	10.33	0.19	15.12	5.27	2.29	0.09	0.71	0.70	1.65	2.39	15
Chile	12.76	0.27	19.78	6.19	3.28	0.03	-0.07	0.84	5.51	2.24	28
Denmark	12.24	0.05	24.32	4.47	1.66	0.09	-0.60	0.59	-2.46	2.65	12
Finland	15.03	0.23	25.72	6.73	4.31	0.43	0.52	1.14	7.04	3.22	21
France	12.01	0.06	20.25	4.63	1.19	0.16	0.17	0.56	5.61	2.73	24
Germany	11.98	0.19	17.33	5.60	2.91	0.38	0.71	0.73	3.34	2.49	34
Hong Kong	13.38	0.27	20.35	6.36	3.15	0.22	0.08	0.83	3.68	2.70	22
Indonesia	16.20	0.15	28.81	6.44	5.10	0.04	0.83	1.79	2.93	14.48	53
Ireland	12.18	0.18	21.46	5.59	2.50	0.31	0.55	0.68	4.30	1.93	25
Italy	11.75	0.29	17.19	6.23	4.18	0.28	0.21	1.01	4.25	2.00	38
Japan	11.58	0.12	23.78	5.29	3.49	0.02	0.22	0.74	4.96	2.25	28
Jordan	7.07	0.43	10.57	5.19	4.03	-0.17	-0.13	1.79	3.81	1.78	14
Korea	16.09	0.17	29.69	6.45	6.37	0.34	0.59	1.01	8.15	3.04	70
Malaysia	12.45	0.07	23.81	5.00	4.38	-0.11	-0.13	0.81	3.70	10.07	42
Mexico	15.44	0.21	24.93	6.63	4.83	0.33	0.23	0.81	8.18	2.93	51
Netherlands	10.52	0.12	15.20	4.71	0.76	0.55	0.23	0.56	8.06	2.49	18
New Zealand	12.02	0.19	19.91	5.66	2.83	0.31	0.25	0.81	3.85	2.64	20
Norway	13.44	0.19	23.85	5.89	3.15	0.26	0.46	0.80	2.44	2.40	22
Philippines	13.41	0.27	24.79	6.71	6.95	0.17	0.43	1.46	5.43	3.08	67
Portugal	9.84	0.16	15.00	5.33	5.26	0.13	0.31	0.85	4.12	1.83	34
Singapore	11.23	0.20	16.45	5.75	4.02	0.19	0.31	0.82	4.15	2.27	36
Spain	12.11	0.23	18.78	5.78	4.27	0.26	0.28	0.91	3.21	2.02	30
Sweden	13.24	0.19	18.77	6.07	4.76	0.63	0.69	0.81	4.93	2.29	53
Switzerland	11.02	0.08	24.30	4.61	1.34	0.08	0.07	0.58	3.97	2.38	14
Taiwan	15.79	0.21	26.09	6.73	4.96	0.32	1.04	1.05	6.30	2.51	59
Thailand	14.82	0.17	28.83	6.36	8.37	0.32	1.86	1.42	3.75	2.99	64
Turkey	23.41	0.05	68.05	7.00	5.83	0.25	0.07	1.24	12.19	3.00	49
United Kingdom	10.25	0.06	17.85	4.21	0.91	0.05	0.05	0.52	6.92	2.40	10
United States	8.73	0.15	13.06	4.74	2.23	0.16	-0.33	0.58	3.78	2.20	13

Note: The parameter estimates are based on the Delay Rejection algorithm for 250,000 iterations and discarding the first 50,000. The table reports the drift of the log prices (ϕ); the mean reversion (κ), the long run mean (θ), and the standard error (σ_v) of the volatility process; the intensity of the jump process (λ); the leverage effect that induces a relationship between volatilities and prices (ρ); the jump size dependence (ρ_j); the parameter ϕ_v of volatility jump size $H_t^v \sim \exp(\phi_v)$; the parameters ϕ_y and σ_y of return jump size $H_t^y \mid H_t^v \sim N(\phi_y + \rho_j H_t^v, \sigma_y^2)$; and the number of jumps detected. Parameters ϕ, θ, σ_v and λ are reported as annualized. The prior distribution of ρ_j is $N(0,4)$.

Table 3.
Performance Comparison of Portfolio Strategies: Maximum Return Objective with Target Volatility

Target Vol.	Static			Volatility-timing			Static vs. Volatility-timing					Volatility-jump-timing			Static vs. Volatility-jump-timing				
	μ	σ	SR	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}
8%	11.20	7.97	0.652	12.27	8.69	0.721	0.8709	132.3	86.3	5.93	3.70	12.36	8.35	0.762	0.9202	158.7	121.9	7.09	5.21
9%	11.49	9.02	0.609	12.78	9.51	0.713	0.8724	151.2	78.9	7.48	3.37	12.94	9.07	0.765	0.9260	177.2	111.5	8.50	4.67
10%	12.42	10.03	0.640	13.78	10.64	0.731	0.8837	167.7	69.3	8.14	2.97	13.93	10.13	0.784	0.9305	194.5	98.8	8.93	4.17
11%	13.31	11.01	0.664	14.64	11.75	0.735	0.8660	186.6	57.5	8.85	2.42	14.83	11.23	0.786	0.9361	211.7	83.4	9.81	3.44
12%	13.92	12.01	0.660	15.30	12.56	0.741	0.8747	201.3	45.0	9.77	1.88	15.54	12.21	0.782	0.9340	227.3	67.1	10.59	2.75
13%	14.46	12.95	0.654	16.08	13.96	0.722	0.8634	217.7	30.4	10.38	1.24	16.34	13.34	0.775	0.9209	241.5	48.3	11.21	1.94
14%	15.07	13.98	0.649	16.78	14.87	0.725	0.8761	231.4	11.7	10.99	0.48	17.03	14.31	0.771	0.9291	255.6	26.0	11.65	1.05
15%	16.04	15.03	0.668	17.65	15.88	0.734	0.8702	246.7	-7.5	11.71	-	17.91	15.48	0.769	0.9264	269.3	4.0	12.36	0.16
16%	16.27	16.01	0.642	18.17	16.90	0.720	0.8759	261.0	-29.2	12.28	-	18.94	16.87	0.767	0.9254	282.6	-20.5	13.23	-

Notes: The table compares the out-of-sample performance of the two dynamic strategies (volatility-timing and volatility-jump) with the static strategy for a range of target volatilities from 8%-16%. The portfolio strategies are implemented using the maximum return objective on 34 weekly equity index returns. The results in each line in the table are based on 1,000 trials using a bootstrap sample of 4,000 returns to estimate unconditional returns and unconditional covariance matrix. For each trial, we compute the realized (next-week) returns, and volatilities gained by the static portfolios and by the volatility-timing and volatility-jump-timing efficient dynamic portfolios. The table reports the average annualized mean return (μ), annualized volatility (σ), and Sharpe ratio (SR) for each strategy; the proportion of trials (p-value) in which each dynamic strategy has higher Sharpe ratio than the corresponding static portfolio; the average annualized performance fees (Δ_γ) an investor with quadratic utility and a degree of risk aversion of γ would be indifference between the static and the dynamic-timing strategies; the break-even transaction cost (τ_γ^{BE}) which is the weekly proportional cost that cancels out the utility advantage of a given dynamic strategy. Δ_γ are expressed in annual basis points, while τ_γ^{BE} are expressed in weekly basic points and are only reported when Δ_γ are positive. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.

Table 4.
Performance Comparison of Portfolio Strategies: Minimum Volatility Objective with Target Return

Target return	Static			Volatility-timing			Static vs. Volatility-timing					Volatility-jump-timing			Static vs. Volatility-jump-timing				
	μ	σ	SR	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}
8%	7.50	2.38	0.631	7.66	2.31	0.720	0.8693	3.9	34.8	0.17	1.37	7.75	2.30	0.761	0.9336	6.2	44.4	0.26	1.71
9%	8.46	3.89	0.632	8.62	3.72	0.705	0.8465	7.5	42.5	0.31	1.68	8.67	3.49	0.765	0.9450	9.8	52.1	0.40	2.04
10%	9.38	5.15	0.656	9.52	4.89	0.719	0.8402	11.6	50.2	0.47	1.98	9.59	4.61	0.779	0.9526	13.9	59.8	0.56	2.36
11%	10.41	6.75	0.653	10.54	6.28	0.723	0.8545	15.5	58.0	0.62	2.25	10.62	5.98	0.773	0.9476	17.8	67.6	0.73	2.61
12%	11.47	8.30	0.659	11.58	7.69	0.726	0.8672	19.6	67.4	0.80	2.62	11.68	7.34	0.774	0.9498	22.1	77.4	0.90	2.99
13%	12.51	10.15	0.641	12.62	9.24	0.716	0.8534	23.9	76.6	1.01	3.03	12.72	8.77	0.766	0.9353	26.2	86.1	1.11	3.33
14%	13.56	11.45	0.660	13.66	10.62	0.721	0.8377	28.6	85.8	1.18	3.32	13.77	10.08	0.771	0.9470	30.9	95.5	1.28	3.75
15%	14.39	12.75	0.658	14.47	11.72	0.723	0.8526	33.5	97.7	1.38	3.77	14.60	11.20	0.768	0.9360	35.8	107.3	1.44	4.19
16%	15.43	14.53	0.649	15.48	13.39	0.708	0.8442	38.2	109.5	1.51	4.39	15.64	12.77	0.755	0.9316	40.6	119.1	1.58	4.91

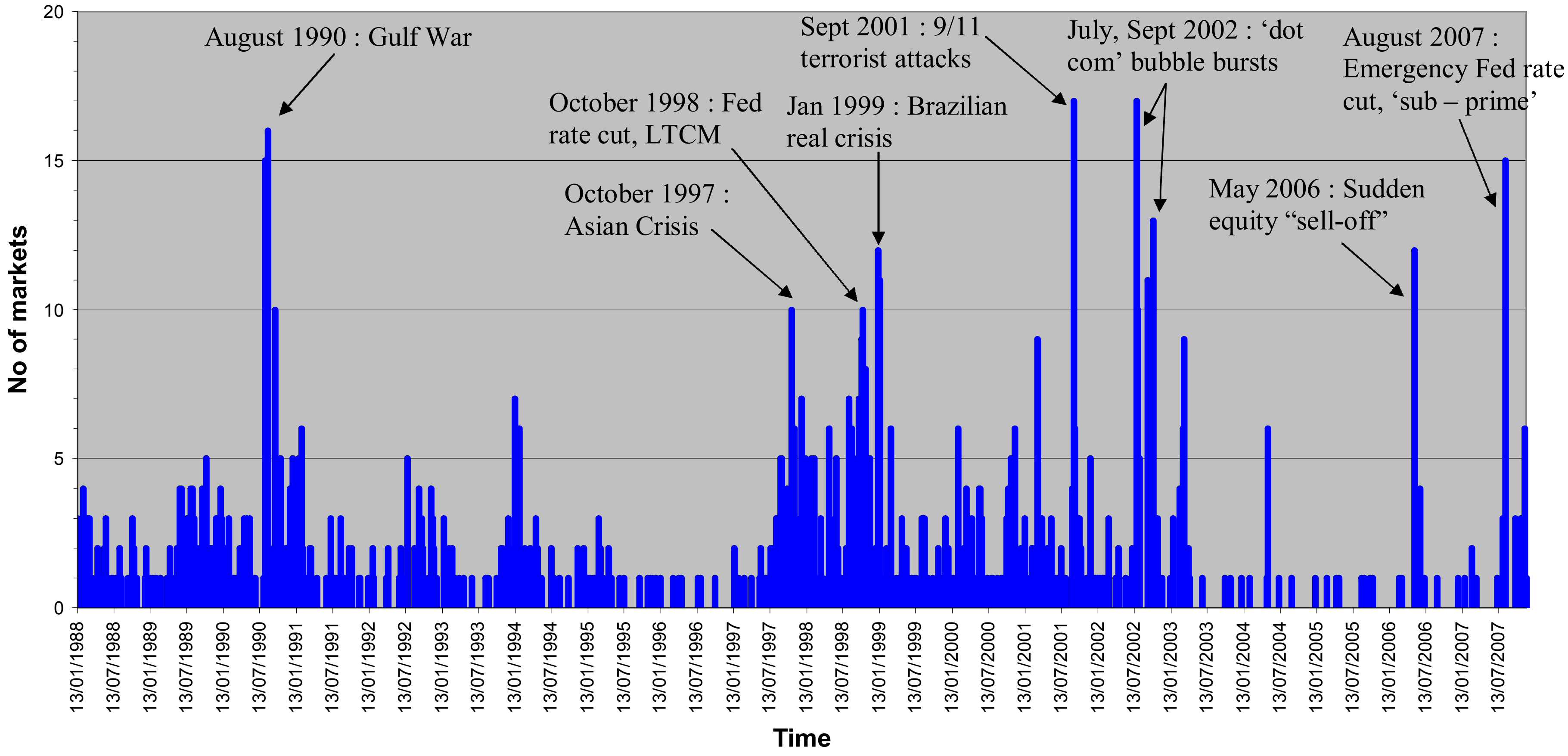
Notes: The table compares the out-of-sample performance of the two dynamic strategies (volatility-timing and volatility-jump) with the static strategy for a range of target returns from 8%-16%. The portfolio strategies are implemented using the minimum volatility objective on 34 weekly equity index returns. The results in each line in the table are based on 1,000 trials using a bootstrap sample of 4,000 returns to estimate unconditional returns and unconditional covariance matrix. For each trial, we compute the realized (next-week) returns, and volatilities gained by the static portfolios and by the volatility-timing and volatility-jump-timing efficient dynamic portfolios. The table reports the average annualized mean return (μ), annualized volatility (σ), and Sharpe ratio (SR) for each strategy; the proportion of trials (p-value) in which each dynamic strategy has higher Sharpe ratio than the corresponding static portfolio; the average annualized performance fees (Δ_γ) an investor with quadratic utility and a degree of risk aversion of γ would be indifference between the static and the dynamic-timing strategies; the break-even transaction cost (τ_γ^{BE}) which is the weekly proportional cost that cancels out the utility advantage of a given dynamic strategy. Δ_γ are expressed in annual basis points, while τ_γ^{BE} are expressed in weekly basic points and are only reported when Δ_γ are positive. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.

Table 5. Year-by-Year Performance Comparison of Portfolio Strategies: Maximum Return Objective with Target Volatility at 12%

Year	No of jumps	Static			Volatility-timing			Static vs. Volatility-timing					Volatility-jump-timing			Static vs. Volatility-jump-timing				
		μ	σ	SR	μ	σ	SR	p-val	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}	μ	σ	SR	p-val	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}
1996	7	21.98	11.99	1.33	23.18	12.32	1.39	0.88	189.6	43.1	9.16	1.78	23.39	12.10	1.44	0.93	218.1	63.8	10.22	2.64
1997	93	-1.24	12.02	-0.60	0.24	12.68	-0.45	0.9	231	53.2	11.22	2.23	0.28	12.59	-0.45	0.91	232.2	54.3	11.01	2.24
1998	141	3.21	12.03	-0.23	4.76	12.86	-0.10	0.90	207.7	47.9	10.14	2.01	4.70	12.99	-0.10	0.89	195.4	41.2	8.22	1.64
1999	78	25.53	12.03	1.62	26.95	12.58	1.66	0.86	182.3	41.0	8.86	1.71	27.30	12.11	1.76	0.96	233.6	81.9	10.96	3.38
2000	74	-6.68	12.02	-1.05	-5.30	12.58	-0.90	0.89	239.0	54.7	11.59	2.29	-4.92	12.11	-0.90	0.93	241.6	63.2	11.34	2.58
2001	80	-2.03	12.02	-0.67	-0.64	12.62	-0.53	0.90	223.5	40.7	10.86	1.69	-0.29	12.15	-0.52	0.90	227.5	49.6	10.72	2.05
2002	91	-1.30	12.02	-0.61	0.15	12.64	-0.46	0.90	228.3	52.9	11.07	2.21	0.44	12.39	-0.45	0.91	237.2	59.5	11.17	2.45
2003	30	32.15	11.97	2.18	33.36	12.51	2.19	0.85	175.7	39.6	8.52	1.66	33.61	12.05	2.29	0.98	239.2	86.4	11.21	3.53
2004	10	25.68	12.00	1.64	27.06	12.38	1.70	0.86	188.1	42.9	9.15	1.79	27.30	12.01	1.77	0.94	230.2	73.6	10.79	3.02
2005	8	22.21	11.98	1.35	23.43	12.37	1.41	0.86	185.1	42.4	8.93	1.76	23.68	12.10	1.46	0.94	209.5	67.3	9.82	2.74
2006	25	25.27	12.02	1.60	26.65	12.49	1.65	0.85	183.1	41.1	8.89	1.71	26.92	12.02	1.74	0.95	226.6	78.4	10.62	3.22
2007	39	22.35	12.02	1.36	23.80	12.62	1.41	0.86	182.8	41.1	8.87	1.72	24.10	12.02	1.51	0.97	233.7	82.7	10.96	3.38

Notes: The table compares the out-of-sample performance of the two dynamic strategies (volatility-timing and volatility-jump) with the static , year-by-year from 1996-2007. The portfolio strategies are implemented using the maximum return objective (with targeted volatility of 12%) on 34 weekly equity index returns. The results in each line in the table are based on 1,000 trials using a bootstrap sample of 4,000 returns to estimate unconditional returns and unconditional covariance matrix. For each trial, we compute the realized (next-week) returns, and volatilities gained by the static portfolios and by the volatility-timing and volatility-jump-timing efficient dynamic portfolios. The table reports the average annualized mean return (μ), annualized volatility (σ), and Sharpe ratio (SR) for each strategy; the proportion of trials (p-value) in which each dynamic strategy has higher Sharpe ratio than the corresponding static portfolio; the average annualized performance fees (Δ_γ) an investor with quadratic utility and a degree of risk aversion of γ would be indifference between the static and the dynamic-timing strategies; the break-even transaction cost (τ_γ^{BE} **)** which is the weekly proportional cost that cancels out the utility advantage of a given dynamic strategy. Δ_γ are expressed in annual basis points, while τ_γ^{BE} are expressed in weekly basic points and are only reported when Δ_γ are positive. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.

Figure 1. Frequency of co-jumps



This figure plots the number of markets out of the 34 stock markets which witness a jump for each week in the sample period January 13, 1988 to November 28, 2007.

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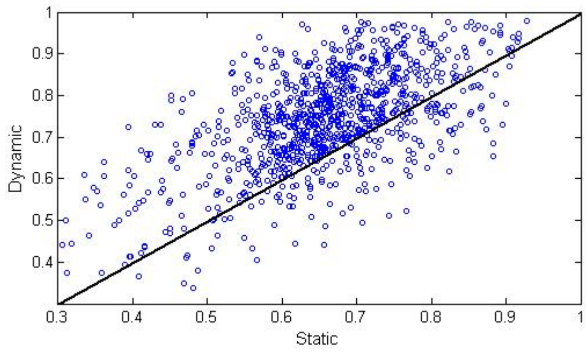
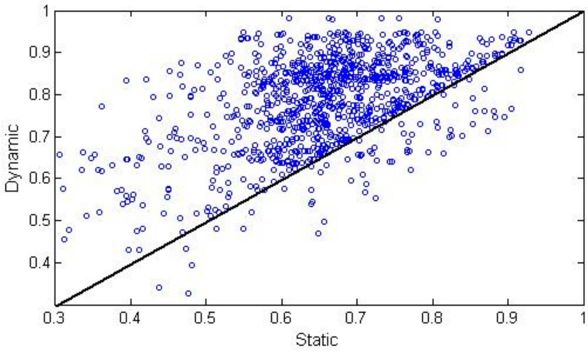
Figure 2.
Sharpe Ratios for the Dynamic vs. Static Strategies

Maximum Return Objective

Volatility-timing

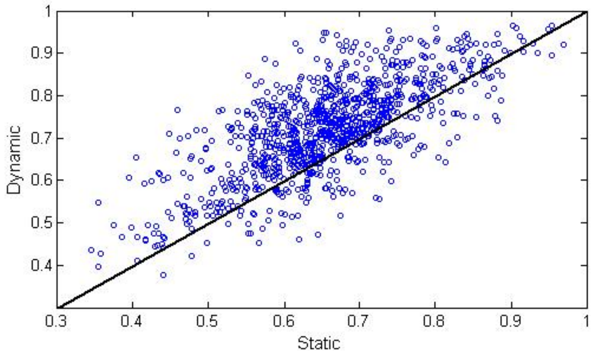


Volatility-jump-timing

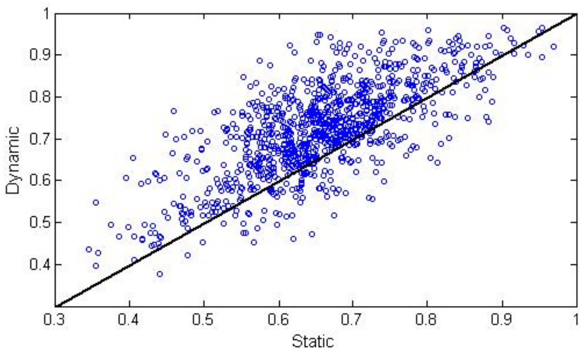


Minimum Volatility Objective

Volatility-timing

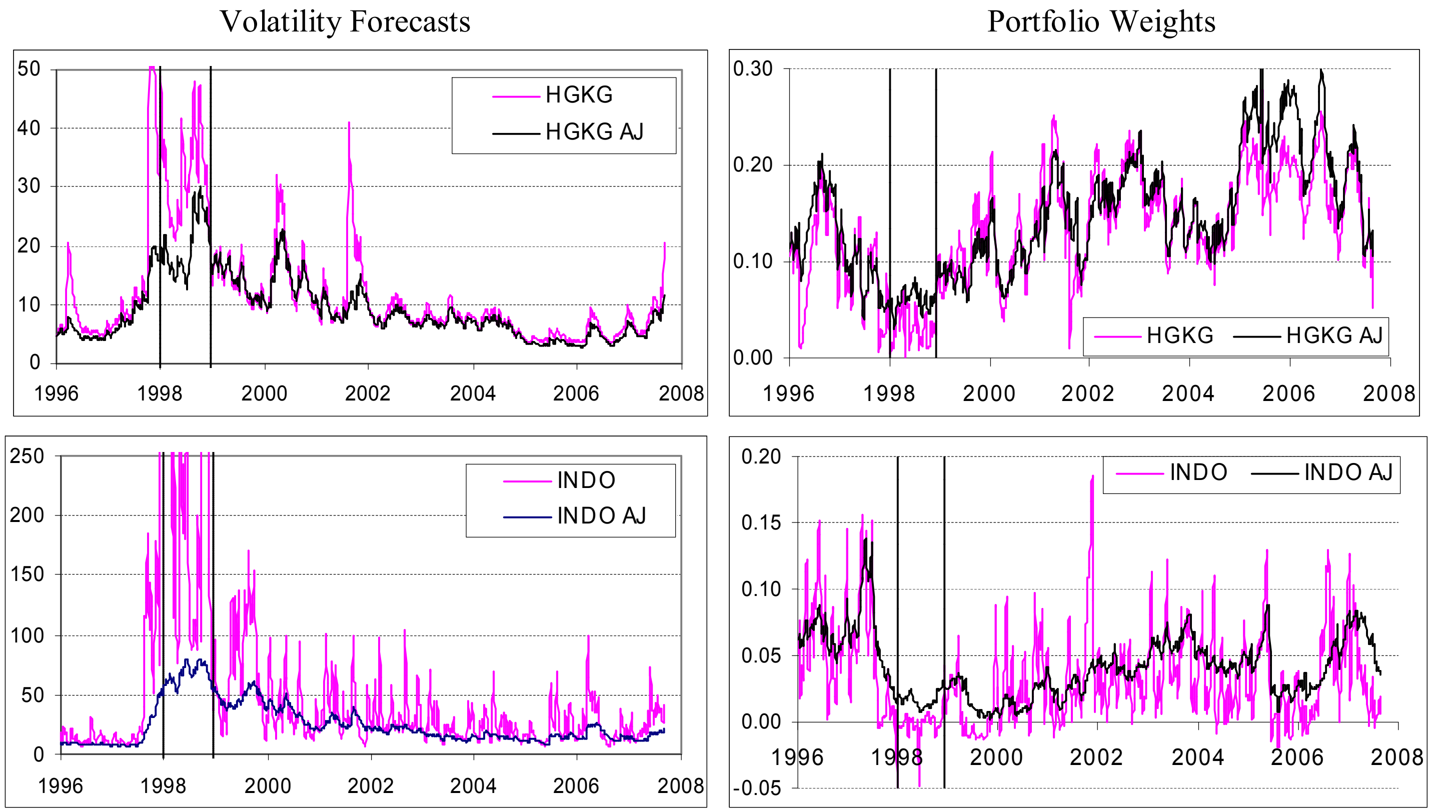


Volatility-jump-timing



Notes: The figure plots the realized Sharpe ratios for the dynamic-timing and static strategies. For the maximum return objective, the target volatility is 12%, whereas for the minimum volatility objective the target return is 10%. We use block bootstrap procedure to generate inputs used in portfolio optimizations. Specifically, from actual returns, we generate a bootstrap sample of 4,000 observations, from which we estimate the vector of unconditional returns and unconditional covariance matrix. For the static strategy, we use the estimated vector of unconditional returns and unconditional covariance matrix to determine the unconditional optimal portfolio. For the dynamic strategies, we use the vector of unconditional returns and the weekly estimated covariance matrix estimate to determine the weekly optimal portfolio. The figure plots the realized Sharpe ratios for each of 1,000 trials of the bootstrap experiment. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.

Figure 3. Comparison of Volatility Forecasts and Portfolio Weights of Raw Returns and Jump-Removed Returns



This figure plots the out-of-sample volatility forecasts (on the left) and the efficient portfolio weights (on the right) of the two dynamic strategies (volatility-timing and volatility-jump-timing). The dynamic strategies are implemented using the maximum return objective with target volatility of 12% on 34 equity index returns. The efficient portfolio weights are based on 1,000 trials using an artificial sample of 4,000 returns to estimate unconditional returns. The countries shown are Hong Kong (HGKG), Indonesia (INDO), Korea (KOR), Malaysia (MAL), and the Philippines (PHLP). "AJ" denotes the results from returns that are adjusted for jumps. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.

Figure 3 (continue).

Volatility Forecasts | Portfolio Weights

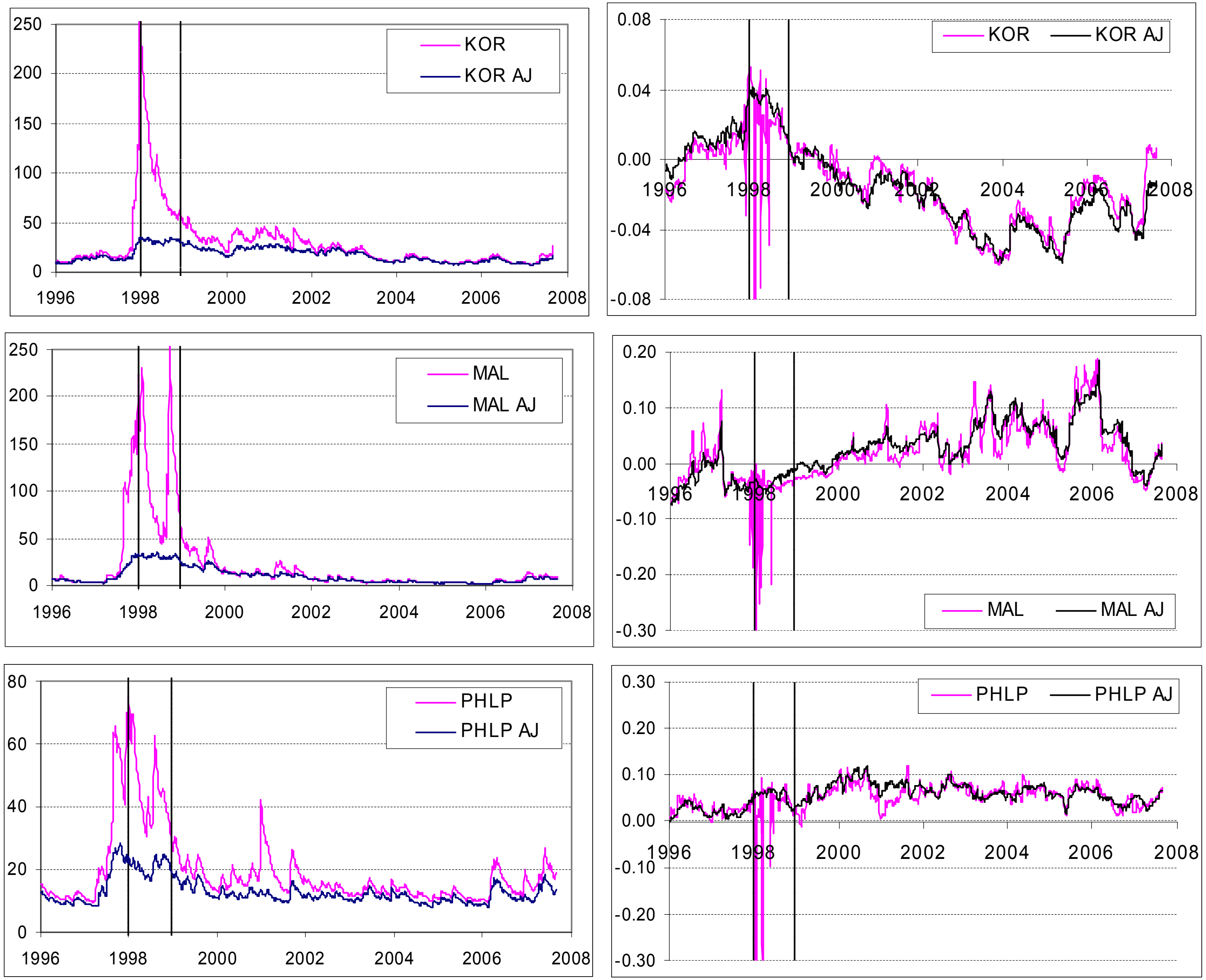


Table 6. Robustness Check on Performance Comparison of Portfolio Strategies with ARJ-GARCH Jump
Maximum Return Objective with Target Volatility

Target Vol	Static			Volatility-timing			Static vs. Static vs. Volatility-timing					Volatility-jump-timing			Static vs. Volatility-jump-timing				
	μ	σ	SR	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}
8%	11.21	7.98	0.653	12.28	8.70	0.722	0.8492	134.3	89.4	6.10	3.90	12.37	8.36	0.763	0.9318	160.8	124.3	7.30	5.39
9%	11.50	9.01	0.612	12.78	9.55	0.710	0.8569	152.4	81.8	7.60	3.58	12.93	9.07	0.765	0.9349	179.2	113.8	8.69	4.82
10%	12.43	10.03	0.641	13.78	10.65	0.731	0.8632	168.2	72.2	8.20	3.13	13.93	10.11	0.785	0.9402	196.5	100.8	9.10	4.34
11%	13.34	11.02	0.666	14.65	11.73	0.737	0.8448	186.4	60.5	8.90	2.61	14.81	11.21	0.786	0.9428	213.8	85.9	9.96	3.57
12%	13.94	12.01	0.661	15.30	12.52	0.743	0.8552	201.6	48.0	9.87	2.10	15.39	12.01	0.782	0.9391	229.4	69.8	10.73	2.93
13%	14.44	12.93	0.653	16.06	13.98	0.720	0.8422	219.2	33.4	10.47	1.39	16.31	13.32	0.774	0.9330	243.6	50.9	11.38	2.11
14%	15.04	13.95	0.648	16.71	14.84	0.722	0.8576	231.8	14.8	11.09	0.68	17.04	14.30	0.772	0.9292	257.7	28.5	11.84	1.24
15%	16.06	15.04	0.669	17.55	15.85	0.729	0.8482	245.6	-5.0	11.71	-	17.91	15.46	0.770	0.9291	271.4	6.0	12.55	0.29
16%	16.32	16.03	0.644	18.16	16.93	0.718	0.8534	263.1	-26.9	12.46	-	18.92	16.85	0.767	0.9273	284.8	-18.5	13.39	-

Notes: The table compares the out-of-sample performance of the two dynamic strategies (volatility-timing and volatility-jump) with the static strategy for a range of target volatilities from 8%-16%. The portfolio strategies are implemented using the maximum return objective on 34 weekly equity index returns. The results in each line in the table are based on 1,000 trials using a bootstrap sample of 4,000 returns to estimate unconditional returns and unconditional covariance matrix. For each trial, we compute the realized (next-week) returns, and volatilities gained by the static portfolios and by the volatility-timing and volatility-jump-timing efficient dynamic portfolios. The table reports the average annualized mean return (μ), annualized volatility (σ), and Sharpe ratio (SR) for each strategy; the proportion of trials (p-value) in which each dynamic strategy has higher Sharpe ratio than the corresponding static portfolio; the average annualized performance fees (Δ_γ) an investor with quadratic utility and a degree of risk aversion of γ would be indifference between the static and the dynamic-timing strategies; the break-even transaction cost (τ_γ^{BE}) which is the weekly proportional cost that cancels out the utility advantage of a given dynamic strategy. Δ_γ are expressed in annual basis points, while τ_γ^{BE} are expressed in weekly basic points and are only reported when Δ_γ are positive. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.

Table 7. Robustness Check on Performance Comparison of Portfolio Strategies with ARJ-GARCH Jump Minimum Volatility Objective with Target Return

Target return	Static Portfolio			Volatility-timing			Static vs. Static vs. Volatility-timing					Volatility-jump-timing			Static vs. Volatility-jump-timing				
	μ	σ	SR	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}	μ	σ	SR	p-value	Δ_1	Δ_{10}	τ_1^{BE}	τ_{10}^{BE}
8%	7.52	2.40	0.634	7.57	2.17	0.725	0.8519	6.3	38.3	0.24	1.46	7.64	2.14	0.767	0.9372	8.0	47.8	0.31	1.80
9%	8.45	3.89	0.630	8.49	3.44	0.723	0.8429	9.3	45.0	0.36	1.70	8.57	3.36	0.765	0.9333	12.7	55.8	0.47	2.12
10%	9.36	5.23	0.642	9.38	4.61	0.734	0.8649	14.4	54.1	0.56	2.07	9.51	4.54	0.773	0.9472	16.8	63.6	0.62	2.41
11%	10.52	7.00	0.646	10.57	6.19	0.739	0.8615	18.1	62.6	0.68	2.32	10.60	5.90	0.780	0.9531	19.9	72.0	0.78	2.71
12%	11.60	8.62	0.650	11.65	7.50	0.753	0.8799	22.5	72.2	0.85	2.71	11.67	7.21	0.786	0.9545	25.0	82.2	0.94	3.15
13%	12.54	10.17	0.643	12.59	8.87	0.742	0.8653	25.7	80.0	1.04	3.09	12.65	8.50	0.782	0.9514	28.8	91.1	1.15	3.44
14%	13.74	11.78	0.657	13.75	10.32	0.751	0.8797	32.4	90.8	1.26	3.47	13.78	9.89	0.787	0.9548	33.9	100.6	1.35	3.90
15%	14.23	12.62	0.652	14.25	11.08	0.744	0.8749	36.6	101.3	1.45	3.85	14.58	10.94	0.784	0.9524	37.8	112.6	1.49	4.33
16%	15.41	14.57	0.646	15.43	12.76	0.739	0.8677	42.0	112.9	1.57	4.44	15.51	12.23	0.778	0.9481	42.8	124.8	1.62	5.06

Notes: The table compares the out-of-sample performance of the two dynamic strategies (volatility-timing and volatility-jump) with the static strategy for a range of target returns from 8%-16%. The portfolio strategies are implemented using the minimum volatility objective on 34 weekly equity index returns. The results in each line in the table are based on 1,000 trials using a bootstrap sample of 4,000 returns to estimate unconditional returns and unconditional covariance matrix. For each trial, we compute the realized (next-week) returns, and volatilities gained by the static portfolios and by the volatility-timing and volatility-jump-timing efficient dynamic portfolios. The table reports the average annualized mean return (μ), annualized volatility (σ), and Sharpe ratio (SR) for each strategy; the proportion of trials (p-value) in which each dynamic strategy has higher Sharpe ratio than the corresponding static portfolio; the average annualized performance fees (Δ_γ) an investor with quadratic utility and a degree of risk aversion of γ would be indifference between the static and the dynamic-timing strategies; the break-even transaction cost (τ_γ^{BE}) which is the weekly proportional cost that cancels out the utility advantage of a given dynamic strategy. Δ_γ are expressed in annual basis points, while τ_γ^{BE} are expressed in weekly basic points and are only reported when Δ_γ are positive. The in-sample period starts from January 13, 1988 to December 27, 1995, and the out-of-sample period begins from January 3, 1996 to November 28, 2007.