# Efficient Estimation of Firm-Specific Betas and its Benefits for Asset Pricing and Portfolio Choice


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# Why Do We Need Good Estimates of Security Betas?

Accurate estimates of firm-specific betas needed for:

1. Capital budgeting
2. Event studies
3. Asset pricing
4. Portfolio optimization
5. Risk management
6. Performance evaluation

# What Is The Problem?

- "Firm-specific betas are difficult to estimate and may well be unstable over time."
  (Campbell, Lettau, Malkiel, and Xu, JF 2001)
- "Given the imprecision of beta estimates for individual stocks, little is lost in omitting them from the cross-section regressions."
  (Fama and French, JF 2008)

# Solution: Bayesian Estimation of Security Betas

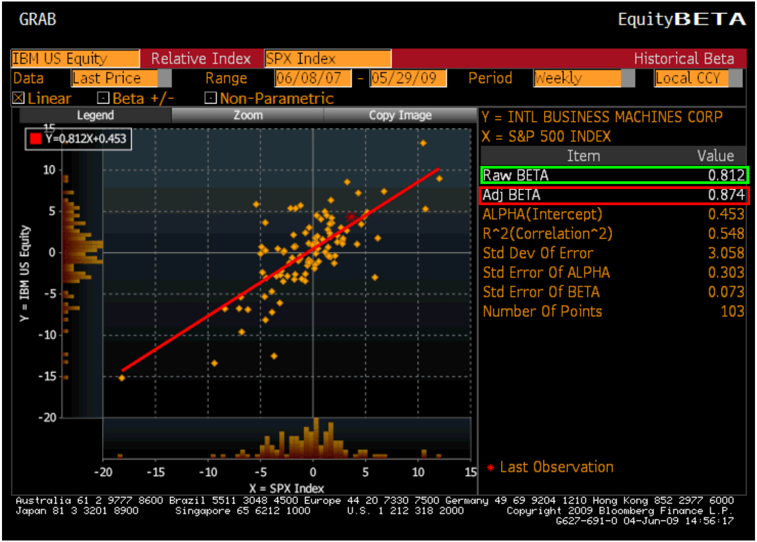
- "Prior cross-sectional information is particularly important in the case of estimating betas of stocks, where the prior information is usually sizeable."
  (Vasicek, JF 1973)

- Classic Vasicek framework:

$$\beta_{i,post} = \phi_i \beta_{i,OLS} + (1 - \phi_i)\, \beta_{i,prior}$$

  where $\beta_{i,prior} = 1$ and $\phi_i$ proportional to precision of OLS estimate and prior distribution

# Bayesian Betas in Practice (I)




Source: Bloomberg

# Bayesian Betas in Practice (II)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ADJUSTED BETA

Bloomberg Financial Definition

Adjusted Beta. An estimate of a security's future beta. Adjusted beta is derived from historical data, but modified by the assumption that a security's beta moves toward the market average of one over time. Adjusted beta is calculated using the following formula:

Adjusted beta = (.67) * Raw beta + (.33) * 1.0



Source: Bloomberg

# What Do We Contribute?

Extend Vasicek model by:

1. Allowing for time variation in firm-specific betas
   - Combine parametric and non-parametric approaches to model changes in beta

2. Shrinking towards theory-based, firm-level prior beta
   - Specify hierarchical Bayesian panel data model to obtain more precise parameter estimates

3. Using daily data to gain efficiency in OLS estimate of beta
   - Use Mixed Data Sampling (MIDAS) to combine different data frequencies and determine optimal weights in estimation

# Main Results

- Combination of OLS and prior betas needed to accurately model dynamics of firm-specific betas
- Panel approach yields more precise estimates of security betas than traditional approach of estimating time series regressions
- Precise beta estimates lead to sharp increase in explanatory power of conditional CAPM
- Minimum variance portfolio constructed using Bayesian beta forecasts outperforms competing approaches out-of-sample

# Existing Methods to Model Time Variation in Betas

Two main approaches to modeling time variation in betas:

1. **Parametric specification: Conditioning variables**
   - Model betas as linear function of conditioning variables (Shanken (1990), Ferson and Harvey (1999))
   - Drawbacks:
     - Set of conditioning information unknown
     - Sudden spikes in betas

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2. **Non-Parametric specification: Data-driven filters**
   - Short-window regressions (Lewellen and Nagel (2006)), Rolling regressions (Fama and French (1997))
   - Drawbacks:
     - Lags true variation in betas
     - Low estimation precision in short window

# Existing Methods to Estimate Betas

Two main approaches to estimating betas:

1. **Group stocks into portfolios** (Fama and MacBeth (1973), Fama and French (1992))
   - Assumes homogeneity within portfolios
   - Limited number of characteristics can be addressed simulateneously

# Existing Methods to Estimate Betas

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2. **Estimate separate time-series regression for each firm** (Brennan, Chordia, Subrahmanyam (1998), Avramov and Chordia (2006))
   - Noisy estimates when time period is short

# Model Specification

Single factor panel data model for excess returns:

$$r_{it} = \alpha_i + \beta_{it-1} r_{Mt} + \epsilon_{it}$$

Bayesian posterior beta is linear combination of time-varying OLS beta and prior beta:

$$\beta_{it} = \phi_{it} b_{OLS,it} + (1 - \phi_{it})\, \beta_{prior,it}$$
$$\phi_{it} = \phi_{0i} + \phi_1 V_{Mt}$$

Weights $\phi_{it}$ are function of realized market variance:

- In volatile times OLS beta captures short run dynamics
- Fundamental prior beta tracks long-term trends in beta

# OLS Beta

1. **OLS (realized) beta**:

$$b_{OLS,it} = \frac{\sum_{\tau=1}^{\tau^{max}} w_{t-\tau} r_{it-\tau}^{(d)} r_{Mt-\tau}^{(d)}}{\sum_{\tau=1}^{\tau^{max}} w_{t-\tau} r_{Mt-\tau}^{(d)} r_{Mt-\tau}^{(d)}} + \frac{\sum_{\tau=1}^{\tau^{max}} w_{t-\tau} r_{it-\tau}^{(d)} r_{Mt-\tau-1}^{(d)}}{\sum_{\tau=1}^{\tau^{max}} w_{t-\tau} r_{Mt-\tau-1}^{(d)} r_{Mt-\tau-1}^{(d)}}$$

- Monthly beta estimated using daily data to increase precision
- Rolling window estimation based on 250 trading days
- Flexible and parsimonious weighting scheme

# Prior Beta

❷ **Prior (fundamental) beta**:

$$\beta_{prior,it} = \delta_0 + \delta_1'[Z_{it} \otimes BC_t]$$

- Informative prior based on conditional asset pricing theory
- Allows relation between beta and firm characteristics to vary with business cycle
- Cross-sectional pooling of $\delta$ parameters to improve accuracy

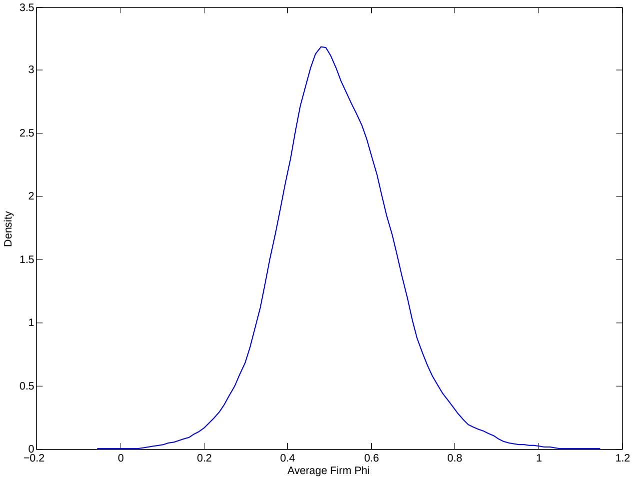
# Why Use Bayesian Methods?

- Capture heterogeneity by specifying hierarchical priors that impose common structure on firm-specific parameters
- Bayesian estimator of firm-specific betas shrinks least squares estimator towards theory-based prior beta
- Hierarchical Bayesian panel structure can accommodate large number of conditioning variables and risk factors
- Easy implementation using MCMC methods to obtain draws from joint posterior density

# Data

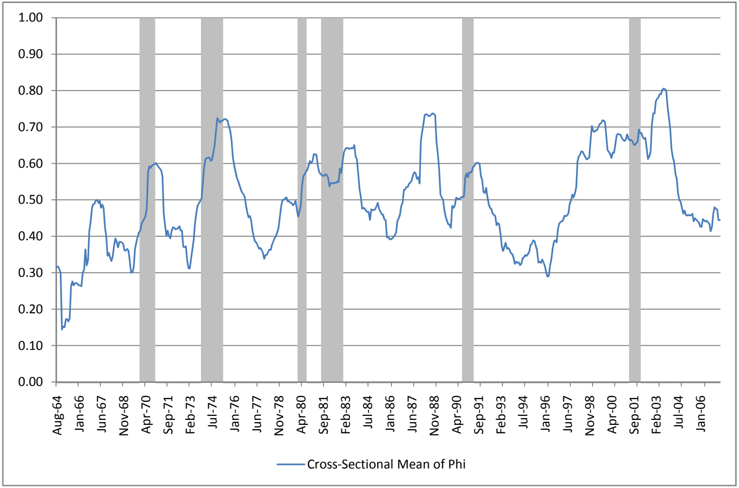
- Daily and monthly stock returns (CRSP)
- 5,017 NYSE/AMEX stocks, July 1964 - December 2006
- Conditioning variables:
  1. Firm characteristics (Gomes, Kogan, and Zhang (2003)): Size, B/M, Momentum
  2. Macroeconomic variables (Ferson and Harvey (1999)): default spread, dividend yield, T-Bill rate, term spread

# Cross-Sectional Variation in Shrinkage Parameter $\phi$



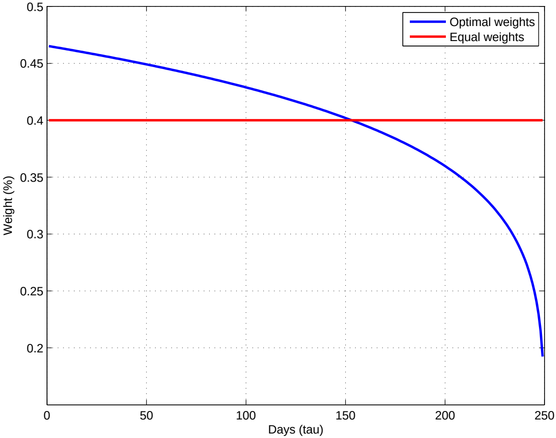

- For average firm, prior beta and OLS beta equally important
- But for some firms one beta much more important than other

# Time Variation in Shrinkage Parameter $\phi$



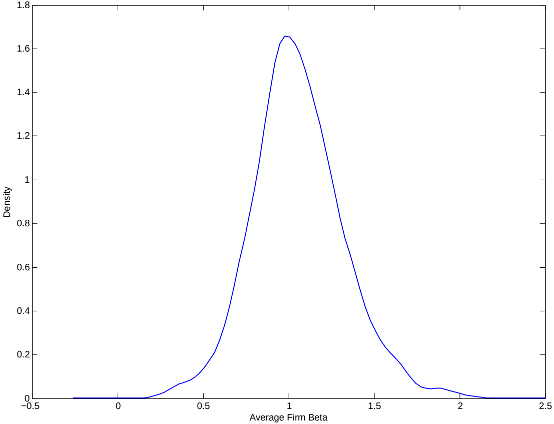

- Substantial time variation in optimal combination of prior and OLS betas
- During turbulent periods, more weight should be given to OLS beta

# Optimal Weighting Scheme for OLS Betas




- Weights reflect trade-off between timeliness and accuracy of OLS beta
- More weight should be given in estimation to most recent days

# Cross-Sectional Distribution of Bayesian Betas

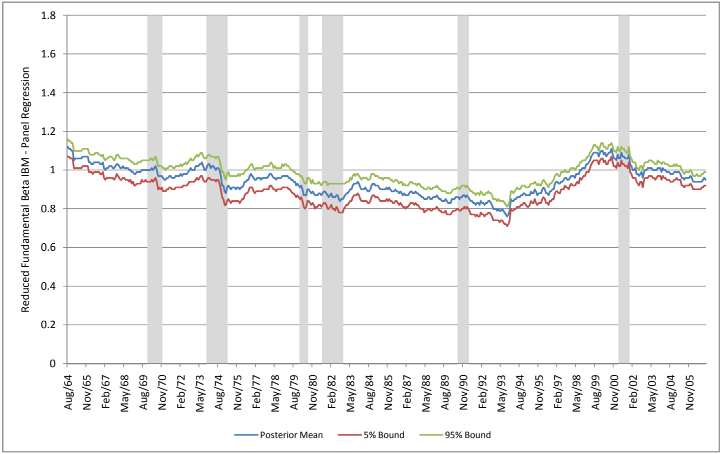



- Cross-sectional distribution of beta centered around one
- Most betas between 0.5 and 1.5, in line with literature

# Beta Estimation Precision

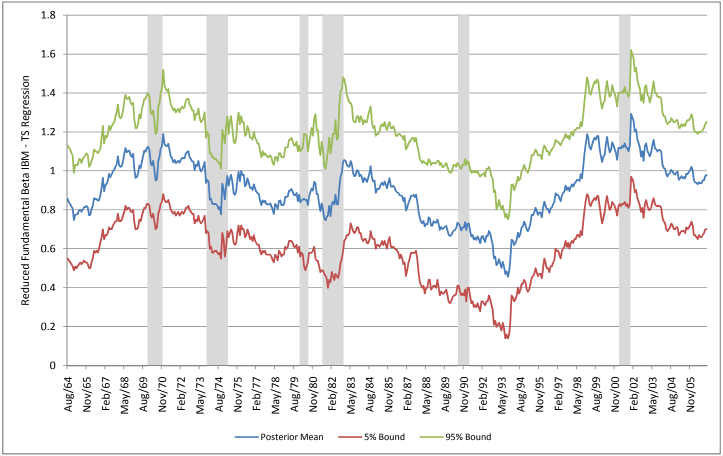
- Compare accuracy of beta estimates from panel model to time series regressions
- Measure estimation precision by computing Bayesian confidence interval for IBM beta
- Consider reduced prior beta specification of Avramov and Chordia (2006)

# Confidence Interval Reduced IBM Beta: Panel Model




- Bayesian panel approach estimates time-varying IBM beta with high precision

# Confidence Interval Reduced IBM Beta: TS Model




- Time series regression produces much wider intervals for IBM beta
- Difference in precision even larger for firms with shorter return history

# Beta Forecasts and Optimal Portfolios (I)

- Markowitz (1952) portfolio theory relies on sample moments
- Difficult to accurately estimate expected returns and covariances when number of assets is large
- Using sample covariance matrix to construct MV portfolios leads to extreme weights and poor out-of-sample performance

# Beta Forecasts and Optimal Portfolios (I)

- Markowitz (1952) portfolio theory relies on sample moments
- Difficult to accurately estimate expected returns and covariances when number of assets is large
- Using sample covariance matrix to construct MV portfolios leads to extreme weights and poor out-of-sample performance

- Many strategies to improve performance of MV portfolios: shrinkage estimators, short-selling constraints, factor structure
- DeMiguel, Garlappi, and Uppal (2007): sophisticated methods do not consistently beat naive $1/N$ rule
- Global minimum variance portfolio does well because no estimates of expected returns needed

# Beta Forecasts and Optimal Portfolios (II)

1. Use Bayesian beta estimates to forecast covariance matrix

$$S_{t+1} = s_{Mt}^2 B_t B_t' + D \tag{1}$$

2. Construct global minimum variance portfolio

$$min \quad w_t' S_{t+1} w_t, \tag{2}$$

$$s.t. \sum_i w_{it} = 1 \tag{3}$$

3. Compare out-of-sample performance to sample covariance matrix, static one-factor structure, and $1/N$ rule

# Beta Forecasts and Optimal Portfolios (III)

| Model | Mean | Std. Dev. | Sharpe Ratio | Short Interest |
|---|---|---|---|---|
| *Panel A: Unconstrained (all stocks)* | | | | |
| Equally weighted ($1/N$) | 8.57 | 15.40 | 0.56 | 0.00 |
| Static beta | 5.24 | 8.50 | 0.61 | -64.64 |
| Bayesian beta | 5.60 | 8.12 | 0.69 | 0.00 |

- Portfolio based on Bayesian beta has lowest out-of-sample volatility

# Beta Forecasts and Optimal Portfolios (IV)

| Model | Mean | Std. Dev. | Sharpe Ratio | Short Interest |
|---|---|---|---|---|
| *Panel A: Unconstrained (all stocks)* | | | | |
| Equally weighted ($1/N$) | 8.57 | 15.40 | 0.56 | 0.00 |
| Static beta | 5.24 | 8.50 | 0.61 | -64.64 |
| Bayesian beta | 5.60 | 8.12 | 0.69 | 0.00 |
| *Panel B: Unconstrained (250 stocks)* | | | | |
| Sample covariance matrix | 4.30 | 15.19 | 0.28 | -144.79 |
| Equally weighted ($1/N$) | 9.46 | 15.93 | 0.59 | 0.00 |
| Static beta | 7.81 | 13.32 | 0.59 | -60.35 |
| Bayesian beta | 5.70 | 8.41 | 0.68 | 0.00 |

- $1/N$ rule performs bad because individual stocks have high idiosyncr. vol.

# Beta Forecasts and Optimal Portfolios (V)

| Model | Mean | Std. Dev. | Sharpe Ratio | Short Interest |
|---|---|---|---|---|
| *Panel A: Unconstrained (all stocks)* | | | | |
| Equally weighted ($1/N$) | 8.57 | 15.40 | 0.56 | 0.00 |
| Static beta | 5.24 | 8.50 | 0.61 | -64.64 |
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| Equally weighted ($1/N$) | 9.46 | 15.93 | 0.59 | 0.00 |
| Static beta | 7.81 | 13.32 | 0.59 | -60.35 |
| Bayesian beta | 5.70 | 8.41 | 0.68 | 0.00 |
| *Panel C: Nonnegativity Constrained (250 stocks)* | | | | |
| Sample covariance matrix | 3.02 | 11.74 | 0.26 | 0.00 |
| Equally weighted ($1/N$) | 9.46 | 15.93 | 0.59 | 0.00 |
| Static beta | 7.55 | 15.60 | 0.48 | 0.00 |
| Bayesian beta | 5.70 | 8.41 | 0.68 | 0.00 |

- Short-selling constraint reduces effect of sampling error in sample covariance matrix but Bayesian beta approach still dominates

# Conclusions

1. Powerful new method for accurately estimating time-varying betas of individual stocks
2. Bayesian betas have predictive power for cross-section of stock returns
3. Bayesian betas increase out-of-sample performance of minimum variance portfolios
4. Approach can be easily extended (more risk factors etc.)
5. Many other potential applications, including performance evaluation, prediction of alphas, and estimation of risk premia