# Dynamic Correlation Hedging in Copula Models for Portfolio Selection

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# Outline

- Motivation and objectives
- The model for asset prices - accounting for extreme asset co-movements through:
  - flexible tail dependence modeling
  - introducing observable factors to drive the dynamics of conditional asset return correlations
- The portfolio choice problem:
  - Market price of risk hedging demands due to increased tail dependence
  - Correlation hedging demands due to observable factors
- Conclusion

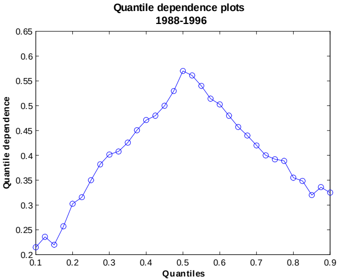
# Asymmetries and downside risk

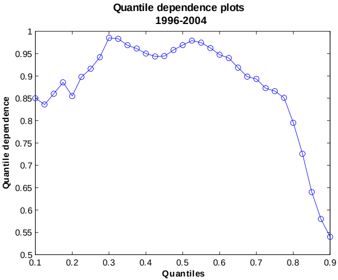
- Probability that assets in a portfolio will jointly decline
  - Correlation? Tail events (extreme moves) ask for different dependence measures
- Asymmetries:
  - Univariate case: skewness
  - Multivariate case: widespread evidence that correlations are higher in extreme market downturns than in extreme market upturns (Longin and Solnik (2001), Ang and Chen (2002), Poon, Rockinger, Tawn (2004))
  - Theoretical justification of this empirical fact: REE model, Ribeiro and Veronesi (2002)
- Portfolio choice implications
  - Beyond mean-variance: investors' sensitivity to downside risk - aversion to extreme negative returns
  - More than myopic behaviour: hedging terms that shift the portfolio composition under extremal dependence

**A 'near' tail dependence measure** (Coles, Currie and Tawn, 1999):

*the probability that one variable exceeds a certain quantile given that the other has exceeded it:*

Plots of quantile dependence for the de-trended log-prices of S&P 500 vs. NASDAQ for the 1988-1996 and 1996-2004 subperiods (weekly data).



**Reasons behind this empirical fact?**

- Can be shown to ensue from the agents' increased uncertainty about the state of the economy

**Implications?**

- Risk management
- Portfolio composition - shift towards the riskless asset when increased dependencies in bad states

# Objectives

Model extreme asset co-movements, driven by latent and observable factors:

- We propose a model that is able to accommodate an extremal dependence structure in two methodologically distinct ways:
  - Through the **stationary distribution** of the process for asset prices (tail dependence)
  - Through a **dynamic conditional correlation** specification, driven by latent and observable factors
- ... that also models in a tractable way the univariate asset return properties
- ... while keeping a continuous time complete market setup for tractable portfolio solutions

# Objectives

Correlation hedging due to latent and observable factors:

- Detect changes in the **portfolio composition**: a shift towards the risk-free asset in turmoil periods of increased dependence in the extremes
- Determine the **loss** in terms of wealth resulting from disregarding dependence during extreme return realizations
- Isolate **correlation hedging demands** due to observable factors that drive dependence between the assets in the portfolio

# The model for stock prices

- **De-trended log prices are represented as an affine function of both latent and observable factors**

$$S_{it} = S_{0t} \exp\left(\alpha_i X_t + \beta_i F_t + k_i t\right) \qquad , i = 1...d$$

where

$X_t$ are latent state variables

$F_t$ are observable factors

both have an underlying diffusion process

- A simple analogy with GBM
- Incorporate thick tails and dependence in extreme realizations in the stationary distribution of the state variable process
- Significance from the perspective of an investor with a long-term investment horizon

# The model for stock prices (cont.)

## Incorporating tail dependence

**Tail dependence** is introduced via the stationary distribution ($q$) of the process for the latent state variables $X_t$ by imposing a certain structure on its drift and diffusion terms

The stationary distribution is modeled using flexible dependence functions (copulas) that allow for separate treatment of the univariate properties of the data (achieving thick tails through the marginals $\widetilde{f}^i\left(x_i|\phi\right)$) and the dependence structure (obtaining tail dependence through the copula $\widetilde{c}\left(x_1, \ldots, x_d|\theta\right)$):

$$q\left(x_1, \ldots, x_d|\theta, \phi\right) \equiv \widetilde{c}\left(x_1, \ldots, x_d|\theta\right) \prod_{i=1}^{d} \widetilde{f}^i\left(x_i|\phi\right)$$

# The model for stock prices (cont.)

## Incorporating tail dependence

Degrees of tail dependence that we consider in our model:

- no tail dependence - Gaussian copula
- symmetric upper and lower tail dependence - Student's t copula
- asymmetric upper and lower tail dependence - symmetrized Joe-Clayton copula

**NB.** In both cases the **univariate distributions** of the risky funds allow for thick tails (NIG distribution)
The univariate processes can replicate stylized return properties like volatility clustering and semi-heavy tails, possibly asymmetric.

# The model for stock prices (cont.)

## The dynamics of conditional correlation

The model also allows for time variations in the conditional correlation between the risky funds, which gives rise to correlation hedging demands. **Conditional correlation** is driven by:

1. latent factors $X$ (with tail dependence): conditional correlation is high during bear markets and in volatile periods
2. observable factors $F$ (proxies for capturing macroeconomic conditions (CFNAI index) and market-wide volatility (the VIX)

- **Benchmark**: constant conditional correlation

# Portfolio choice in the presence of extremal dependence

In a complete market, the portfolio holdings in risky assets $\alpha_t$ can be decomposed into:

- **Mean-variance term** (MV)
- **Intertemporal hedging demands** for hedging stochastic changes in the interest rate (IRH) and the market price of risk (MPRH), expressed in terms of conditional expectations of the state variables $Y_t = (X_t, F_t)$ and their Malliavin derivatives

$$\alpha_t = MV\left(Y_t, \omega_t\right) + IRH\left(Y_t, \omega_t\right) + MPRH\left(Y_t, \omega_t\right)$$

- **Effect of tail dependence/conditional correlation:** in the MPR hedge term through the process of the market price of risk and its sensitivity to shocks in the Brownians driving uncertainty of the risky funds.

# Correlation hedging demands

- The market price of risk hedging term is driven by the **sensitivity** of the latent factors $X$ and the observables factors $F$ underlying the stock price **to uncertainty shocks** (i.e. via their Malliavin derivatives)
- If conditional correlation is driven exclusively by certain factors, observable or not, we can explicitly **isolate correlation hedging demands** due to these factors
  - Here: correlation hedgin gdemands due to a macroeconomic factor and a market-wide volatility factor

# The evolution of portfolio hedging terms: in-sample

## Total MPR hedging demands

The portfolio holdings in one asset induce hedging demands for the rest of the assets, as they are exposed to the same sources of risk.

- Induced hedging demands under dynamic conditional correlation are always lower than those under constant conditional correlation
- They are also lower for tail dependent latent risk factors than for tail independent ones

- When the investment horizon falls within a relatively calm period, tail dependence does not impact portfolio MPR hedging terms (or slightly so if conditional correlation is constrained to be constant)
- During a turmoil period tail dependence generally lowers the MPR hedging demands (and more so if conditional correlation is constrained to be constant)

# Correlation or tail dependence hedging effect: out-of-sample

**CRRA or HARA investor with degrees of relative risk aversion of 5 and 10, investment horizon of 5 years**

- If dynamics in conditional correlation are accounted for in the state variable process, incorporating tail dependence still lowers the hedging demands
- If tail dependence is accounted for in the state variable process, incorporating dynamics in the conditional correlation also generally leads to lower hedging demands

-> *both channels of modeling dependence (invariant or conditional) cannot be considered as substitutes in terms of portfolio hedging effects*

- The correlation hedging effect of the macroeconomic factor substantially overweights that of the market-wide volatility factor

# The cost of disregarding dependence between extreme realizations

Given that the true data generating process has both tail dependence and dynamic correlation, modeled with observable factors, we determine **the economic cost for the investor of disregarding dependence between extremes**

- It is larger when the investor disregards both tail dependence and dynamics in conditional correlation
- It is negligible if the investor disregards only the asymmetric nature of tail dependence (higher dependence in downturns than in upturns) and alternatively considers symmetric dependence in the extremes
- It increases for growing degree of dependence in the extremes
- It decreases with increasing levels of risk aversion for all utility specifications considered
- It increases with the investment horizon

# The cost of disregarding dependence between extreme realizations

The Certainty Equivalent Cost is given in cents per dollar. Investment horizon is 5 years.

Panel A. The cost of disregarding tail dependence

| | (Gaussian alternative, DCC) | | | (Gaussian alternative, CCC) | | |
|---|---|---|---|---|---|---|
| | HARA | CRRA | HARA | HARA | CRRA | HARA |
| | b=-0.2 | b=0 | b=0.2 | b=-0.2 | b=0 | b=0.2 |
| $\gamma = 2$ | 1.3153 | 1.5158 | 1.7162 | 3.2467 | 3.8692 | 4.4916 |
| $\gamma = 6$ | 0.3912 | 0.4619 | 0.5326 | 0.4602 | 0.6562 | 0.8523 |
| $\gamma = 10$ | 0.1902 | 0.2327 | 0.2751 | 0.0000 | 0.0507 | 0.1664 |

**NB.** The true data generating process has both tail dependence and dynamic correlation

# The cost of disregarding dependence between extreme realizations

The Certainty Equivalent Cost is given in cents per dollar. Investment horizon is 5 years.

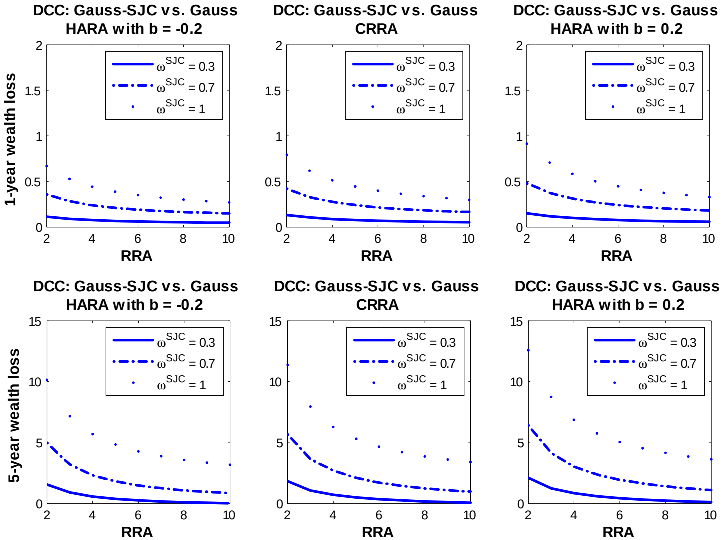
Panel B. The cost of disregarding asymmetric tail dependence

| | (Student's $t$ alternative, DCC) | | | (Student's $t$ alternative, CCC) | | |
|---|---|---|---|---|---|---|
| | HARA | CRRA | HARA | HARA | CRRA | HARA |
| | b=-0.2 | b=0 | b=0.2 | b=-0.2 | b=0 | b=0.2 |
| $\gamma = 2$ | 0.1886 | 0.1696 | 0.1506 | 0.5891 | 0.6486 | 0.7081 |
| $\gamma = 6$ | 0.4259 | 0.4403 | 0.4546 | 0.3960 | 0.4260 | 0.4559 |
| $\gamma = 10$ | 0.3999 | 0.4106 | 0.4213 | 0.3224 | 0.3411 | 0.3598 |

**NB.** The true data generating process has both tail dependence and dynamic correlation

# The cost of disregarding tail dependence

## ...increases with growing dependence in the extremes



# The cost of disregarding dynamics of conditional correlation

Given that the true data generating process has both tail dependence and dynamic correlation, modeled with observable factors, we determine **the economic cost for the investor of disregarding dynamics in conditional correlation**

- It is lower than the cost of disregarding extreme dependencies for an otherwise similar investor
- It increases with increased levels of conditional correlation
- It decreases with increasing levels of risk aversion for all utility specifications considered
- It increases with the investment horizon

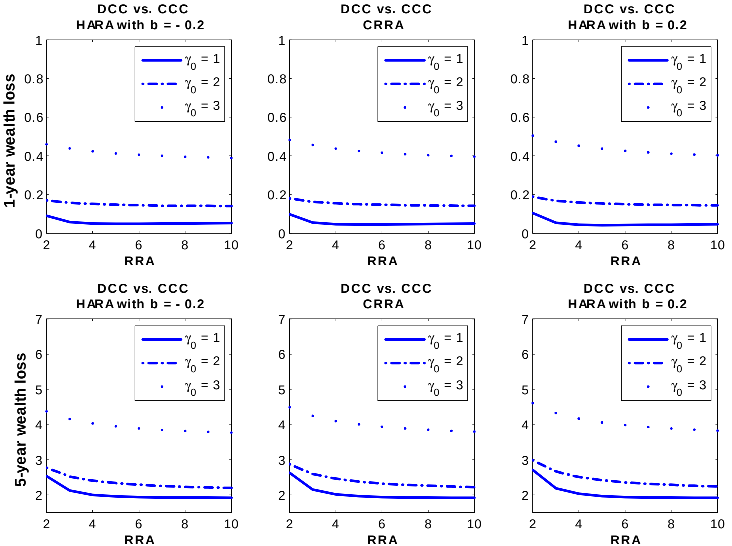
# The cost of disregarding dynamics of conditional correlation

The Certainty Equivalent Cost is given in cents per dollar. Investment horizon is 5 years.

| *Panel C.* The cost of disregarding DCC | | | |
|---|---|---|---|
| | HARA, $b = -0.2$ | CRRA | HARA, $b = 0.2$ |
| $\gamma = 2$ | 2.3054 | 2.4039 | 2.5024 |
| $\gamma = 6$ | 1.7983 | 1.8216 | 1.8449 |
| $\gamma = 10$ | 1.7289 | 1.7419 | 1.7549 |

**NB.** The true data generating process has both tail dependence and dynamic correlation

## ...increases with increased levels of conditional correlation



# Conclusion

What we have found so far:

- The portfolio solution methodology allows us to isolate:
  - correlation hedging demands due to observable factors
  - the impact of tail dependence on market price of risk hedging terms
- Correlation hedging demands and intertemporal demands due to high level of tail dependence have a distinct impact on the optimal portfolio behavior:
  - both in terms of portfolio composition
  - and economic significance

# Further developments

What we are currently working on:

- Introduce stochastic variations in the dependence structure of the process for the latent state variables, driven by observable factors
- Set of observables: market level and market-wide stockastic volatility
- Out-of-sample performance of observed vs. latent factor models for portfolio allocation