Dynamic Correlation Hedging in Copula Models for Portfolio Selection

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Abstract

In this paper we address the issue of modeling extreme asset co-movements and their implications for the hedging demands of a dynamic portfolio. We propose a model that is able to accommodate an extremal dependence structure through the stationary distribution of the state variables underlying the asset price process, as well as through a dynamic conditional correlation specification, driven by latent and observable factors. With this we aim at replicating the stylized fact of increased dependence during extreme market downturns, rising market-wide volatility, or worsening macroeconomic conditions. The model we propose accounts for stylized properties of asset returns in terms of univariate tail behavior as well as varying forms of dependence in the extremes , while keeping a continuous time complete market setup for a tractable portfolio solution. The paper further concentrates on the portfolio implications of those stylized facts. We isolate the intertemporal hedging demands, including those for correlation risk due to stochastic changes in the factors. Thus, we are able to analyze separately the impact of tail dependence through the unconditional distribution of the underlying state variables and that of conditional correlation on the portfolio holdings. We find that both correlation hedging demands and intertemporal hedges due to increased tail dependence have distinct portfolio implications and cannot act as substitutes to each other. As well, there are substantial economic costs for disregarding both the dynamics of conditional correlation and the dependence in the extremes.

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1 Summary

The correlation structure of world equity markets is not constant over time, but is highly time varying. A number of studies have addressed this issue, as well as the driving factors behind this time variation. Based on data from the last 150 years, Goetzmann et al. (2005) find that correlations between equity returns vary substantially over time and achieve their highest levels during periods characterized by highly integrated financial markets. As well peaks in correlations and not only volatility can be attributed to major market crashes, as for example the Crash of 1929. Longin and Solnik (1995) study shifts in global equity markets correlation structure and reject the hypothesis of constant correlations among international stock markets. Moreover, they find evidence that correlations increase during highly volatile periods. Using Extreme Value Theory, Longin and Solnik (2001) find that international stock markets tend to be highly correlated during extreme market downturns than during extreme market upturns, establishing a pattern of asymmetric dependence. Another strand of literature connects the variability of stock return correlations to the phase of the business cycle. Ledoit et al. (2003) and Erb et al. (1994) show that correlations are time-varying and depend on the state of the economy, tending to be higher during periods of recession. Similar evidence is brought forward by Moskowitz (2003) who links time variation of volatilities and covariances to NBER recessions.

The evidence of highly varying conditional correlations on equity markets has motivated us to propose a continuous time process for asset prices that incorporates the above mentioned stylized facts in two distinct ways. First, we allow for tail dependence between extreme realizations of asset returns by explicitly modeling the stationary distribution of underlying latent variables using copula functions that incorporate dependence in the left or the right tail. This allows us to obtain higher dependence when markets experience downturns than during upward moves. Further, in order to exploit the conditional correlation structure of asset returns, we propose a specification for modeling correlation dynamics using observed factors, including macroeconomic and market volatility factors. With those we aim at capturing the above mentioned features of asset returns, and namely the fact that correlations increase during extreme market downside moves, hectic periods and recessionary states of the economy.

This framework buys us the flexibility to remain in a complete market setting in which portfolio solutions can be obtained for a wide class of investor utility functions. We are able to undertake a standard portfolio solution methodology which allows us to obtain in closed form up to numerical integration the optimal portfolio components in terms of mean-variance demand and intertemporal hedging demands. For the case where we model conditional correlation as a function of observed factors, we are also able to isolate the hedging demands for correlation risk, due to stochastic changes in the factors. We use the solution for the optimal portfolio allocation in order to address the following issues:

a) We test whether the implications of allowing for tail dependence through the stationary distribution and for dynamic conditional correlation on the optimal portfolio hedging demands are similar in magnitude and direction. As those distributional assumptions aim at replicating the same stylized behavior, it is interesting to see whether the portfolio effects will share this similarity. For an in-sample market timing exercise along realized paths of the state variables over a 20-year investment horizon and two risky funds, we find that allowing for dynamic conditional correlation generally drives up the intertemporal hedging demands, while allowing for tail dependence in the stationary distribution diminishes them. There is also a distinction in the portfolio composition between the risky funds: in the presence of dynamic conditional correlation the spread between the hedging demands for the two funds increases, while in the presence of tail dependence it decreases, bringing about smaller hedging components in absolute value for the two funds. Those effects become more important when increasing the investment horizon.

b) We further investigate the evolution of the correlation hedging demands implied by the observable factors. Using a factor to capture market-wide volatility and another one to account for macroeconomic conditions, we find that the total correlation demands due to those factors are generally negative throughout the period we consider. The impact of the macroeconomic factor is more significant and directs the behavior of the hedging demands.

c) We test whether results are sensitive to the particular choice of investment period. We consider two sub-periods that differ in the level of stock market volatility and macroeconomic conditions, and we consider an investor with investment horizon set at the end of each of these sub-periods. We find that for a relatively calm period with almost no extreme events towards its end the impact of tail dependence disappears once we allow for a data generating process that incorporates dynamics in the conditional correlation behavior. To the contrary, for a hectic period with declining macroeconomic conditions and a number of extreme events, especially towards its end, the importance of modeling tail dependence for the optimal hedging demand cannot be overwritten by allowing for dynamically varying correlations.

d) We further test the economic importance of considering dynamic conditional correlation or tail dependence using the concept of the certainty equivalent cost and find substantial utility loss due to disregarding either form of dependence, which increases with the investment horizon and for low levels of the agent's relative risk aversion. As well, we find substantial utility loss for disregarding dependence between extreme realizations, even when dynamic conditional correlation has already been accounted for, and vice versa. We also compare different dynamic conditional correlation specifications that take into account or not observable factors and we find that there is utility loss related to disregarding observable factors, especially factors related to macroeconomic conditions.

e) As well we study the sensitivity of the optimal hedging behavior for different levels of average correlation and find higher hedging demands for high correlation levels, when the impact of stochastic changes in conditional correlation on investor's utility is expected to be the highest. This finding is confirmed by the certainty equivalent cost of disregarding dynamic conditional correlation: the utility loss increases for higher levels of average correlation. Alternatively, we study the impact of disregarding tail dependence for varying levels of tail dependence coefficients in the data generating process and find that there are far more significant costs of disregarding dependence between extreme realizations when its level increases, even when dynamic conditional correlation is already taken into account.

The portfolio solution methodology that we consider allows us to identify the intertemporal hedging demands that arise from the need to hedge against changes in the stochastic investment opportunity set, and separate them from the mean-variance component. As well, we can solve under general utility preferences, that are not constrained to the CRRA case.