Valuation of non-traded risk factors such as longevity risk and inflation risk

Frank de Jong

Tilburg University and Netspar

Inquire meeting Zürich, March 31, 2008

Motivation

- New accounting and supervisory rules call for market valuation of pension and insurance liabilities
- Use market interest rates to discount nominal liabilities
  - risk-free government bond rates or interbank swap rates
- This approach is rooted in "replication" argument: value of liability is value of portfolio of financial assets that exactly replicates the payoff pattern
- For price indexed liabilities, real interest rates can be used

Problems

- Many pension plans aim at wage indexation
- For wage indexed liabilities, replication by financial market instruments is not obvious:
  - There are no wage-indexed market instruments $\rightarrow$
  - no exact replicating portfolio can be constructed
- Valuation of liabilities using replication argument not possible
- Insurance companies face related valuation problems
  - longevity risk
  - other non-hedgeable insurance risks

Netspar

Suggested alternative discount rates

- Expected return on assets
  - this is not well-founded and makes value of liabilities depend on the asset mix
- Expected return on 'best' (but imperfect) hedge portfolio
  - comes close to the right procedure, but leaves the idiosyncratic (unhedged) risk unpriced

Approaches for valuation in incomplete markets

- Super-replicating portfolio
  - find portfolio that always gives payoff at least as big as the contingent claim
  - not very useful here because wage risk is in principle unbounded
- Good-deal bounds (Cochrane and Saá-Requejo (2000))
  - exclude strategies with 'too good' risk-return tradeoff
  - bounds will be fairly wide for wage risk case (see later)
- Utility-based approach (e.g. Hodges and Neuberger (1989), Henderson (2002), Pelsser (2005))
  - value of portfolio of financial assets that gives the same expected utility as the fully wage-indexed pension
- Market clearing price of risk for 'completing' instrument (e.g. Sangvinatsos and Wachter (2005))

Valuation of contingent claims

- Workhorse of asset pricing theory is the Pricing Kernel
  - interpretation as Stochastic Discount Factor (SDF)
- Value of contingent claim is expectation of product of SDF and payoff stream

$$P_0 = \mathrm{E}\left[\sum_{t=1}^{T} M_t X_t\right]$$

- Pricing Kernel is spanned by the returns on the available financial assets
  - only contingent claims that can be replicated by a trading strategy in existing financial assets can be priced in this way
- SDF can be generalized for non-spanned claims, but value is not unique in that case

Utility maximization and portfolio choice

- Agents maximize expected utility of wealth at time horizon T given initial wealth W_0 and pricing kernel M_T

$$\max_{W_T} \mathrm{E}\left[U(W_T)\right] \quad \text{s.t.} \quad \mathrm{E}[W_T M_T] = W_0$$

- If markets are complete, optimal wealth solution can be found by Cox-Huang technique

$$W_T^* = U'^{-1}\left(\ell M_T\right)$$

- In a complete market, the optimal wealth can be generated by a trading strategy in financial assets

Incomplete markets

- Now assume markets are not complete
- Terminal wealth is a product of hedgeable wealth W_{1T} and unhedgeable component W_{2T}
  - with $W_{2,0} = 1$ and $\mathrm{E}[W_{2,T}] = 1$ as normalization
- Maximization problem then becomes

$$\max_{W_{1T}} \mathrm{E}\left[U(W_{1T}W_{2T})\right] \quad \text{s.t.} \quad \mathrm{E}[W_{1T}M_{1T}] = W_0$$

Netspar

Equivalent utility

- Define the expected utility of complete markets problem as

$$I(W_0) = \mathsf{E}\left[U(W_{1T}^*)\right]$$

With background risk, the expected utility is

$$J(W_0) = \mathsf{E}\left[U(\widehat{W}_{1T} W_{2T})\right]$$

- To obtain the same expected utility as in complete markets case, an additional fraction of initial wealth π has to be invested, with

$$I(W_0) = J(\pi W_0)$$

- For any utility function $U(W)$ which exhibits risk aversion, $\pi > 1$

A model for wage risk (1)

- Uncertainty is generated by stock prices, interest rates and wage level

$$
\begin{aligned}
dS/S &= \mu_S dt + \sigma_S dZ_S \\
dr &= \kappa(\bar{r} - r)dt + \sigma_r dZ_r \\
dY/Y &= g dt + \theta_S dZ_S + \theta_r dZ_r + \theta_w dZ_w
\end{aligned}
$$

- Components of real wage growth:
  - deterministic trend growth g
  - exposures to interest rate θ_r and stock price θ_S
  - idiosyncratic component with volatility θ_w

A model for wage risk (2)

- Market incompleteness: wage risk cannot be hedged perfectly
  - idiosyncratic wage risk $\theta_w dZ_w$ not correlated with market instruments

Pricing kernel

The pricing kernel is

$$dM/M = -rdt - \lambda' dZ - \lambda_w dZ_w$$

where dZ contains the 'spanned' risk factors dZ_S and dZ_r; dZ_w is the unspanned wage risk factor

The traded assets in the economy are all priced by the pricing kernel for the spanned risks

$$dM_1/M_1 = -rdt - \lambda' dZ$$

Good-deal bounds

- Assume absolute value of Sharpe ratio of wage-linked assets $|\lambda_w|$ is bounded by A
- Discount rate to be applied to the expected payoff of a wage linked bond is bounded between $(r - g - A\theta_w, r - g + A\theta_w)$
- With A equal to equity Sharpe ratio 0.2, discount rate is in the range $(0.6\%, 2.4\%)$
- For a wage-indexed pension liability of 100 million euro with a duration of 20 years, this implies a wide range of present values between 62.23 and 88.72 million
- Wide range, not very useful

Utility based valuation

- Investors are interested in standard of living
  - wealth divided by aggregate wage index, Y_t

$$\max \mathrm{E}\left[U(W_T/Y_T)\right], \text{ s.t. } \mathrm{E}\left[W_T M_T\right] = W_0$$

- Assume CRRA utility function

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

  where γ is the coefficient of relative risk aversion

- Then the relation between equivalent initial wealth in the incomplete market case and complete market case

$$\pi = \exp\left(\tfrac{1}{2}\gamma\theta_w^2 T\right)$$

- An additional investment of $\frac{1}{2}\gamma\theta_w^2$ per period is needed to compensate for the unhedged wage risk

Discount rate

- This additional initial wealth can be translated to a lower discount rate for wage-linked pension claims
  - denote r_T the yield on a T period real bond, then the appropriate discount rate on a wage-linked claim is

$$r_T^w = r_T - g + \theta'\lambda - \frac{1}{2}\gamma\theta_w^2$$

- The differences are
  - a correction for the expected growth of the real wage g
  - a correction for the correlation with stock returns and interest rates
  - a compensation for the idiosyncratic risk
- The implied market price of wage risk is $\lambda_w = -\frac{1}{2}\gamma\theta_w$

Netspar

Relation to optimal portfolio

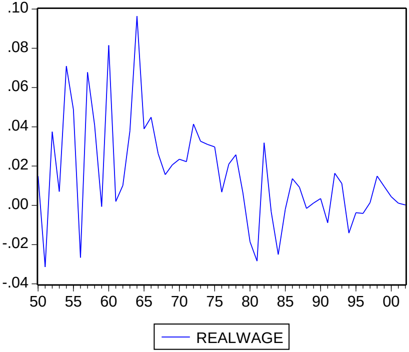
- The optimal portfolio strategy is

$$x^* = \frac{1}{\gamma}\left(\sigma\sigma'\right)^{-1}\sigma\lambda + \left(1-\frac{1}{\gamma}\right)\left(\sigma\sigma'\right)^{-1}\sigma\left[\theta - \sigma_r B(T)e_r\right]$$

  where λ are the market prices of risk and σ is a matrix of exposure of financial instruments to the risk factors
- Can be seen as a weighted average of a speculative portfolio and the 'best' hedge portfolio of the real wage risk
- The discount rate on the wage linked pension equals the expected return on the 'best' replicating portfolio, minus the corrections for the expected wage growth and the idiosyncratic risk

Calibration

- Parameters are calibrated using real wage growth data from the Netherlands, 1950-2002



Calibration (2)

- Idiosyncratic wage risk is 2.5% per annum, but wage shocks are somewhat serially correlated
  - after correcting for serial correlation we find $\theta_w = 4.5\%$
  - We consider a fairly risk averse investor with $\gamma = 5$
  - correction for market incompleteness thus is $-\frac{1}{2}\gamma\theta_w^2 = -0.5\%$
  - implied market price of wage risk is $-\frac{1}{2}\gamma\theta_w = -0.11$
- Correlation between interest rates and wages is negligible, exposure to stock returns is significant but small
  - we find $\theta_S = 0.0030$ with $\sigma_S = 0.20$ and $\lambda_S = \frac{0.04}{0.20} = 0.2$
  - hence $\theta_S\lambda_S = 0.06\%$

Netspar

Discount rates for wage indexed claims

- Assume a real interest rate equal to 3% and an expected real wage growth of 1.5%, then the discount rate for unconditionally wage indexed pension liabilities is

$$r^w = 3\% - 1.5\% - 0.5\% = 1\%$$

- The value of a 100 million ABO with 20 years duration is

$$P_0 = \frac{100}{(1+0.01)^{20}} = 82$$

- For comparison, discounting the expected payoff at the real interest rate would give

$$P_0 = \frac{100(1+0.015)^{20}}{(1+0.03)^{20}} = 74.6$$

implying a difference of about 10% in the Funding Ratio

Calibrations for US data

- Vector Autoregression (VAR) for six variables
- Long run variances and covariances implied from VAR
- Estimated risk premium on wage linked bond
  - stock market risk premium: 45 basis points
  - inflation risk premium: negligible
  - interest rate risk premium: 30 basis points
  - idiosyncratic wage risk premium: -15 basis points (at $\gamma = 5$)

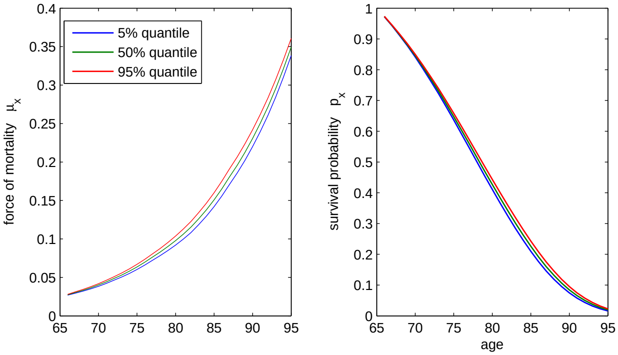
Calibrations for US data

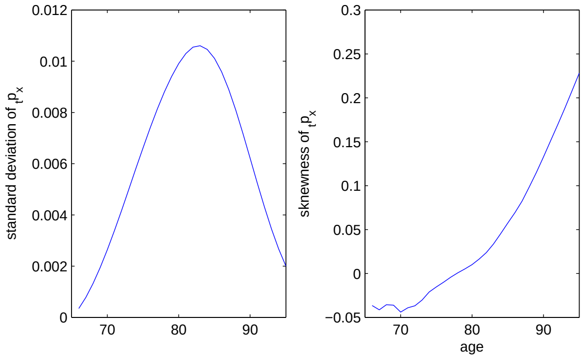
	θ	β	λ	risk premium
wage	0.0240		−0.058	−0.14
stock return	0.0226	0.178	0.20	0.45
inflation	−0.0031	−0.058	−0.15	0.05
interest rate	−0.0207	−0.248	−0.15	0.31

Netspar

Longevity risk

- Cui (2008) applies the equivalent utility principle to the pricing of longevity risk
- Longevity is increasing, but there is uncertainty about future trends





- Pensions, life insurances and annuities are sensitive to longevity risk: uncertain payoff
- Longevity bonds, with coupon linked to survival rates, can hedge this risk
- But how to value such bonds?

Netspar

Equivalent Utility pricing

- Minimal acceptable price of an annuity for the insurer
  - $\mathrm{E}[S_t] + P_t =$ price of one annuity installment
  - $S_t =$ survival fraction
  - $P_t =$ required premium for unhedgeable risk

$$\mathrm{E}\left[U(W_t + \mathrm{E}[S_t] + P_t - S_t)\right] = \mathrm{E}\left[U(W_t)\right]$$

- Translation to discount rates

$$\frac{1}{(1 + R_a + R_p)^t} = \mathrm{E}[S_t] + P_t$$

R_a is annuity rate without longevity risk, R_p is risk premium

Longevity risk premiums

- N insured people
- $W_0 = 100$ insurance firm equity capital
- CRRA utility with $\gamma = 5$

maturity	$N = 100$	$N = 1000$
5	0.00	0
10	0.00	-0.01
15	0.00	-0.03
20	-0.01	-0.05
25	-0.01	-0.06
30	0.00	-0.04
35	0.00	-0.01

Note: risk premium in percent

Conclusion

- Payoff of pension liabilities and other contingent claims often imply incomplete markets
- Valuation based on equivalent utility principle leads to easy adjustment to pricing kernel
- For uniform background risks such as real wage growth, simple adjustment of discount rates can be applied
  - about 0.5% lower discount rate for wage-linked liabilities
  - implies higher values than discounting expected payoff by market interest rates (about 10% for stylized example)