Asset Return Volatility, High-Frequency Data, and the New Financial Econometrics

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Who Uses Volatility Models, and Why?

- Asset pricing
- Portfolio allocation (incl. direct vol positions)
- Risk management (incl. hedging)

Financial Asset Return Data

- Volatility clustering
- Fat tails
- Convergence to normality under temporal aggregation

Generation I: “GARCH Volatility”

Background:

“The Nobel Memorial Prize for Robert F. Engle,”
Scandinavian Journal of Economics, 2004, in press.

Measuring and Forecasting
Financial Market Volatilities and Correlations
New York: W.W. Norton, 2005.

GARCH Process

$$r_t | \Omega_{t-1} \sim N(0,\ h_t)$$

$$h_t = \omega + \alpha\ r_{t-1}^2 + \beta\ h_{t-1}$$

Basic Structure and Properties

Time variation in volatility and prediction-error variance

Unconditional symmetry and leptokurtosis

Convergence to normality under temporal aggregation

ARMA representation in squares

GARCH(1,1) and exponential smoothing

Easy estimation and testing

Variations

Asymmetric response and the leverage effect

Volatility components, Long memory, Regime switching

Fat-tailed conditional densities

GARCH-M and time-varying risk premia

Multivariate

Onward...

• Volatility from parametric models

• Volatility from options prices

• Volatility from direct indicators

$$r_t^2$$

$$|r_t|$$

Useful, but problems remain...

Generation II: Realized Volatility

Estimate volatility by summing intra-period squared returns

Important early work:

- French, Schwert & Stambaugh (1987)
- Schwert (1989, 1990)

New Developments

- Provide rigorous foundations
- Direct characterization of marginal and conditional distributions
- Multivariate analysis
- Direct modeling and forecasting

Plan

- Theory
- Data
- Statics: the marginal distribution of volatility
- Dynamics: the conditional distribution of volatility
- The distribution of standardized returns
- Modeling and Forecasting
- New developments

Theory

$$dp_t = \sigma_t \, dW_t$$

$$r_{(m),t} \equiv p_t - p_{t-1/m} = \int_0^{1/m} \sigma_{t+\tau} dW_{t+\tau} , \; t = 1/m, 2/m, \ldots$$

$$\sigma^2_{t,h} \equiv \int_0^h \sigma^2_{t+\tau} d\tau$$

$$plim_{m\to\infty} \Sigma_{j=1,..,mh} r^2_{(m),t+j/m} = \sigma^2_{t,h}$$

Extensions: multivariate, jumps

Some Background

(1) "The Distribution of Realized Exchange Rate Volatility," *Journal of the American Statistical Association*, 96, 42-55, 2001.

(2) "The Distribution of Realized Stock Return Volatility," *Journal of Financial Economics*, 2001

(3) "Exchange Rate Returns Scaled by Realized Volatility are (Nearly) Gaussian," *Multinational Finance Journal*, 4, 159-179, 2000.

(4) "Modeling and Forecasting Realized Exchange Rate Volatility," *Econometrica*, 71, 579-626, 2003.

(5) "Parametric and Nonparametric Volatility Measurement," in L.P. Hansen and Y. Aït-Sahalia (eds.), *Handbook of Financial Econometrics*, 2005, in press.

Data

Construction of 5-minute DM/$ and Yen/$ returns...

- Average of log bid and log ask, interpolated to 5-minute
- Exclude weekends
- Exclude fixed and variable holidays
- Exclude days with data feed shutdown

Construction of Daily Realized Volatilities and Correlations

$$vard_t \equiv \Sigma_{j=1,..,288} (\Delta logD_{(288),t-1+j/m})^2$$

$$vary_t \equiv \Sigma_{j=1,..,288} (\Delta logY_{(288),t-1+j/m})^2$$

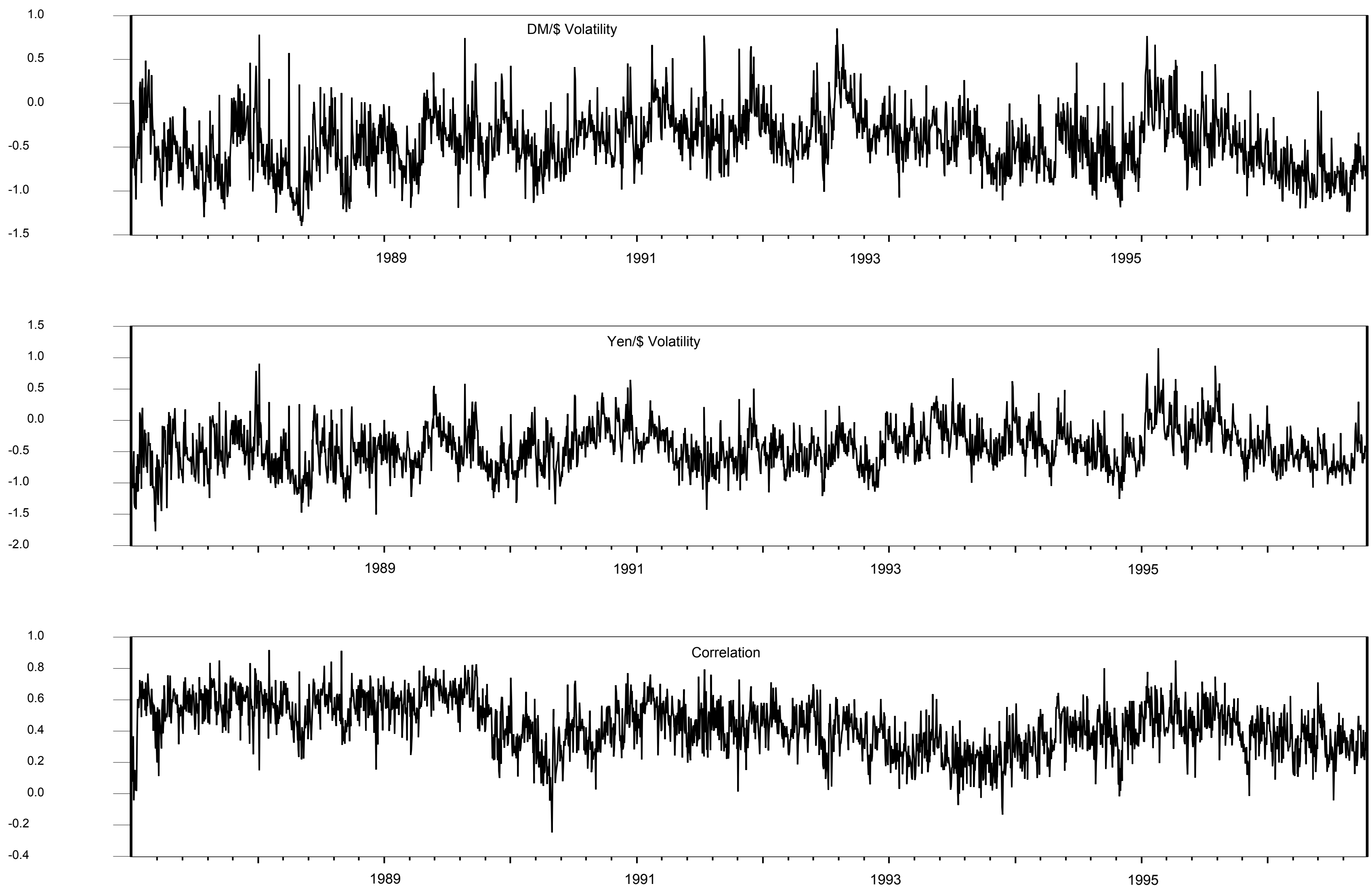
$$cov_t \equiv \Sigma_{j=1,..,288} \Delta logD_{(288),t-1+j/m} \cdot \Delta logY_{(288),t-1+j/m}$$

$$stdd_t \equiv vard_t^{1/2}, stdy_t \equiv vary_t^{1/2}$$

$$lstdd_t \equiv ½ \cdot log(vard_t), lstdy_t \equiv ½ \cdot log(vary_t)$$

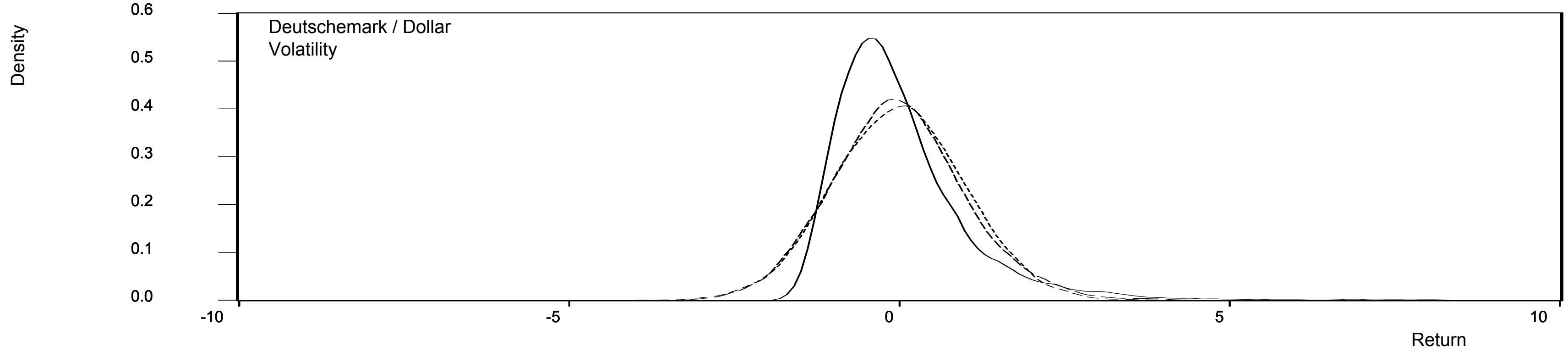
$$corr_t \equiv cov_t/(stdd_t \cdot stdy_t)$$

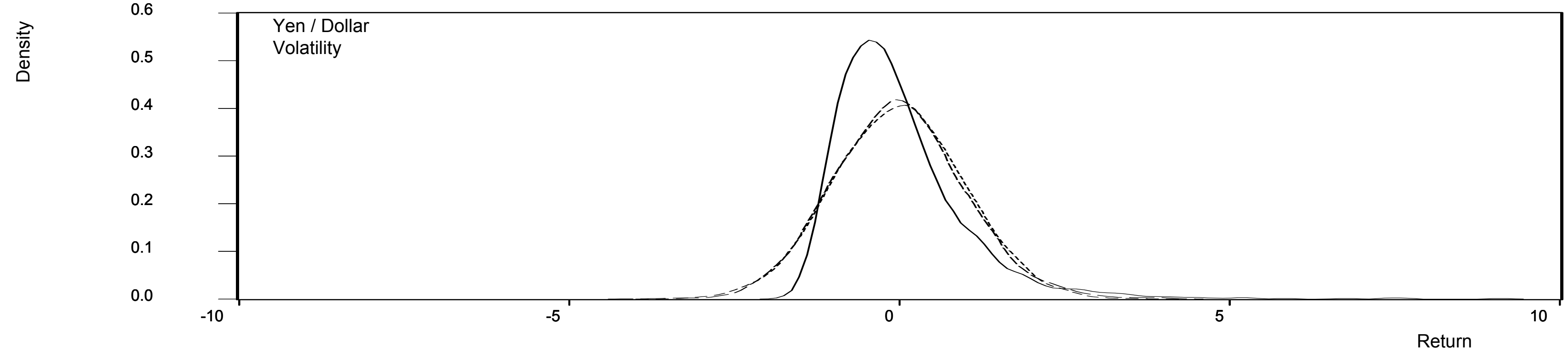
Realized Volatilities and Correlations

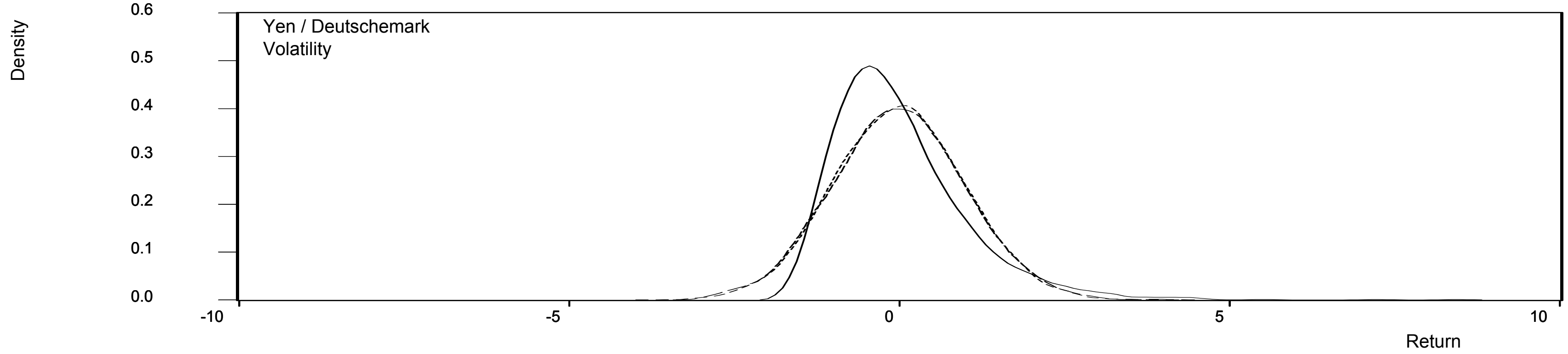


The Distribution of Volatility is Lognormal

Distributions of Realized Volatilities and Correlation





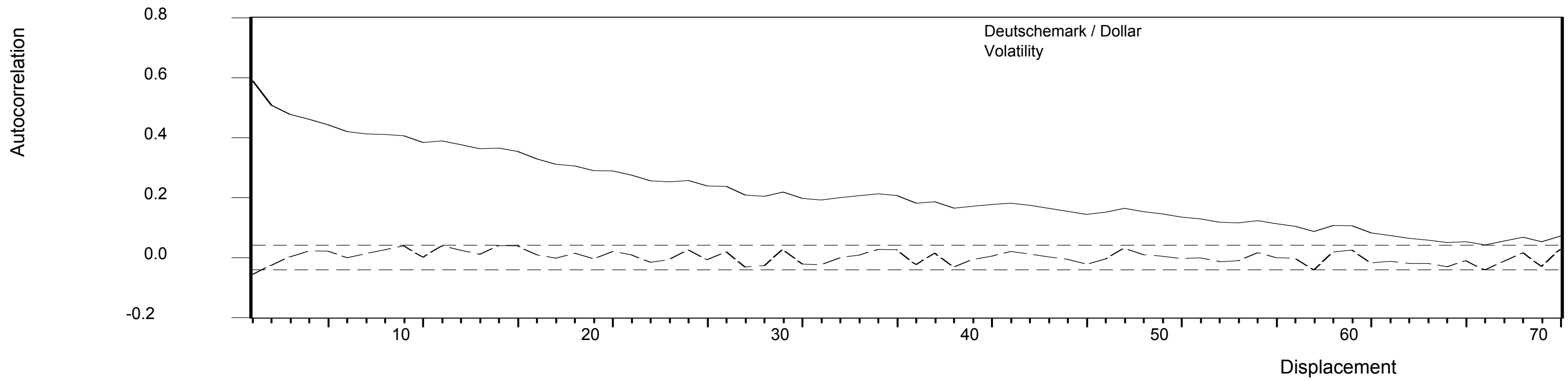


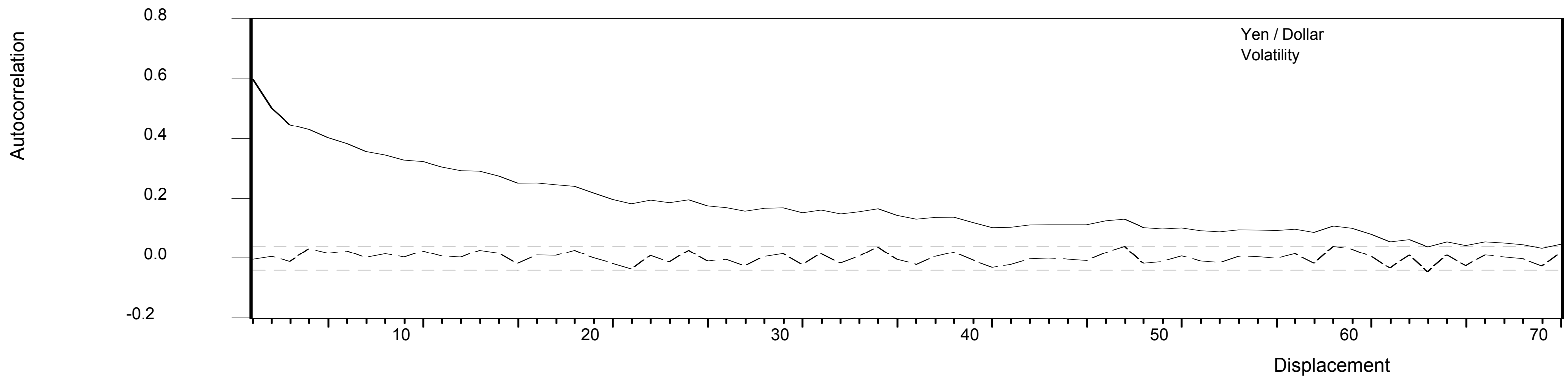
The Dynamics of Realized Volatility are Highly Persistent

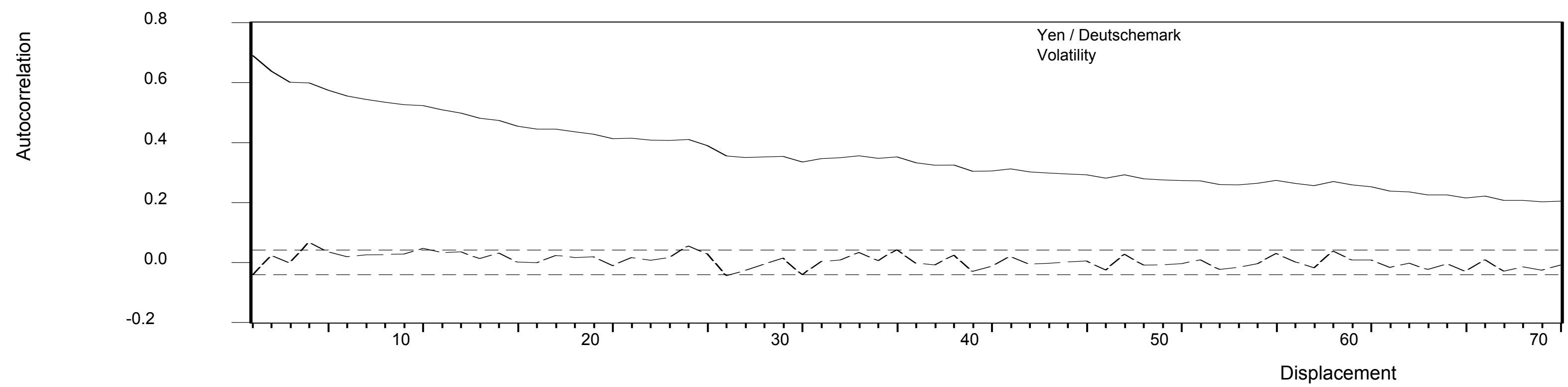
No Unit Roots, but Clear Long-Memory

	$lstdd_t$	$lstdy_t$	$corr_t$
ADF	-6.370	-7.817	-5.589
$\hat{d}$	0.421	0.448	0.423

Autocorrelation Functions







Volatility Forecasts From Long-Memory Models

In-sample: 1986-1996, out-of-Sample: 1997-1999

- VAR-RV: $A(L)(1-L)^{.4}(\sigma_t - \mu) = \epsilon_t$

- RiskMetrics: $\sigma_t^2 = .94\sigma_{t-1}^2 + .06 r_t^2$

- GARCH(1,1): $r_t \mid \Omega_{t-1} \sim N(0, \sigma_t^2)$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2$$

Forecast Evaluation Regressions for Realized Volatilities
Out-of-Sample, One-Day-Ahead

	b_0	b_1 (VAR-RV)	b_2 (Other)	R^2
DM/$				
VAR-RV	0.02 (.05)	0.99 (.09)	-	.25
RiskMetrics	0.22 (.04)	-	0.63 (.08)	.10
GARCH	0.05 (.06)	-	0.85 (.10)	.10
VAR-RV + RiskMetrics	0.02 (.05)	0.98 (.13)	0.01 (.11)	.25
VAR-RV +GARCH	0.02 (.06)	0.98 (.13)	0.02 (.16)	.25

Standardized Returns are Approximately Gaussian

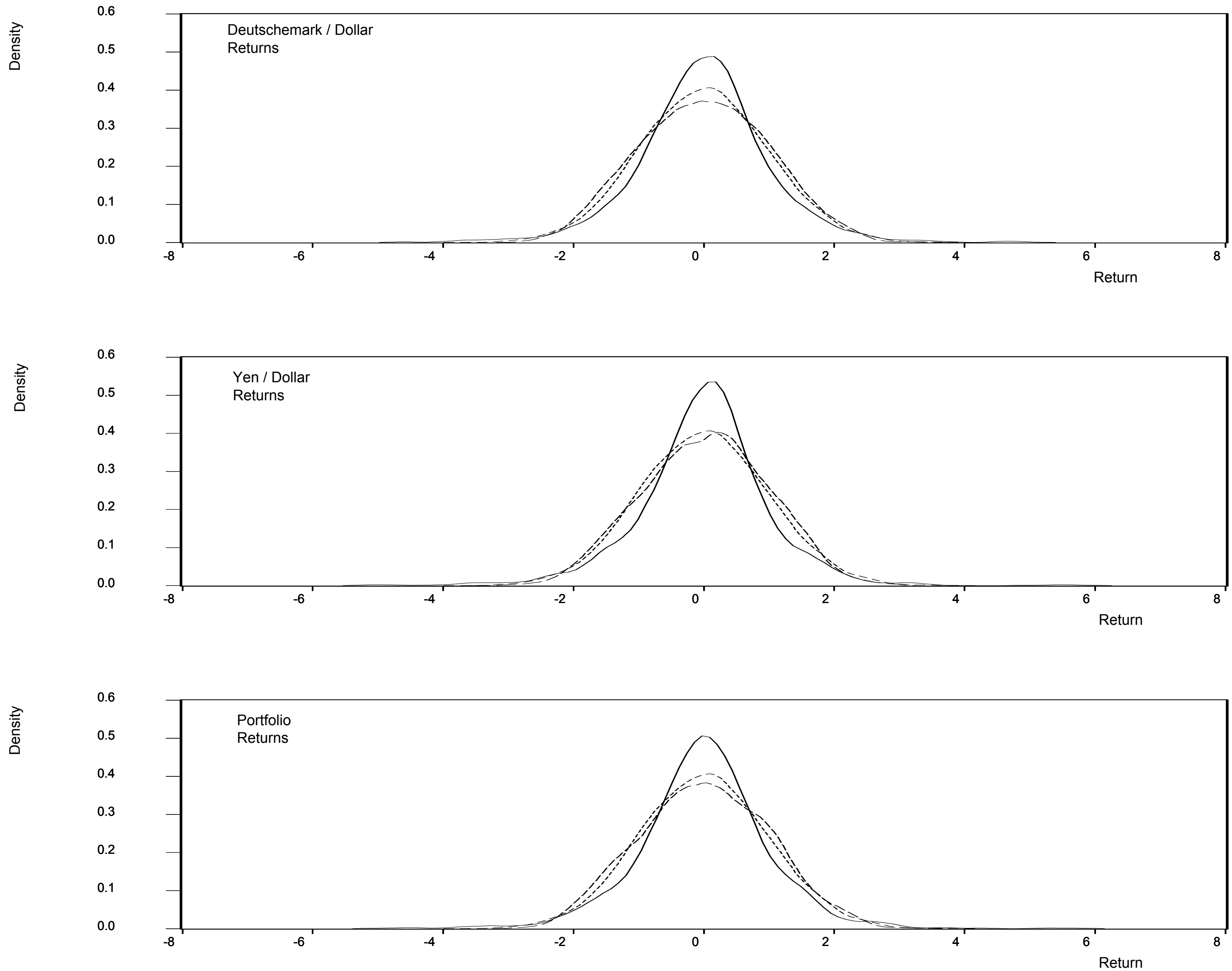
Unstandardized Returns

$$r_t = \sigma_t \, \varepsilon_t$$

Standardized Returns

$$\varepsilon_t = \frac{r_t}{\sigma_t}$$

Return Distributions



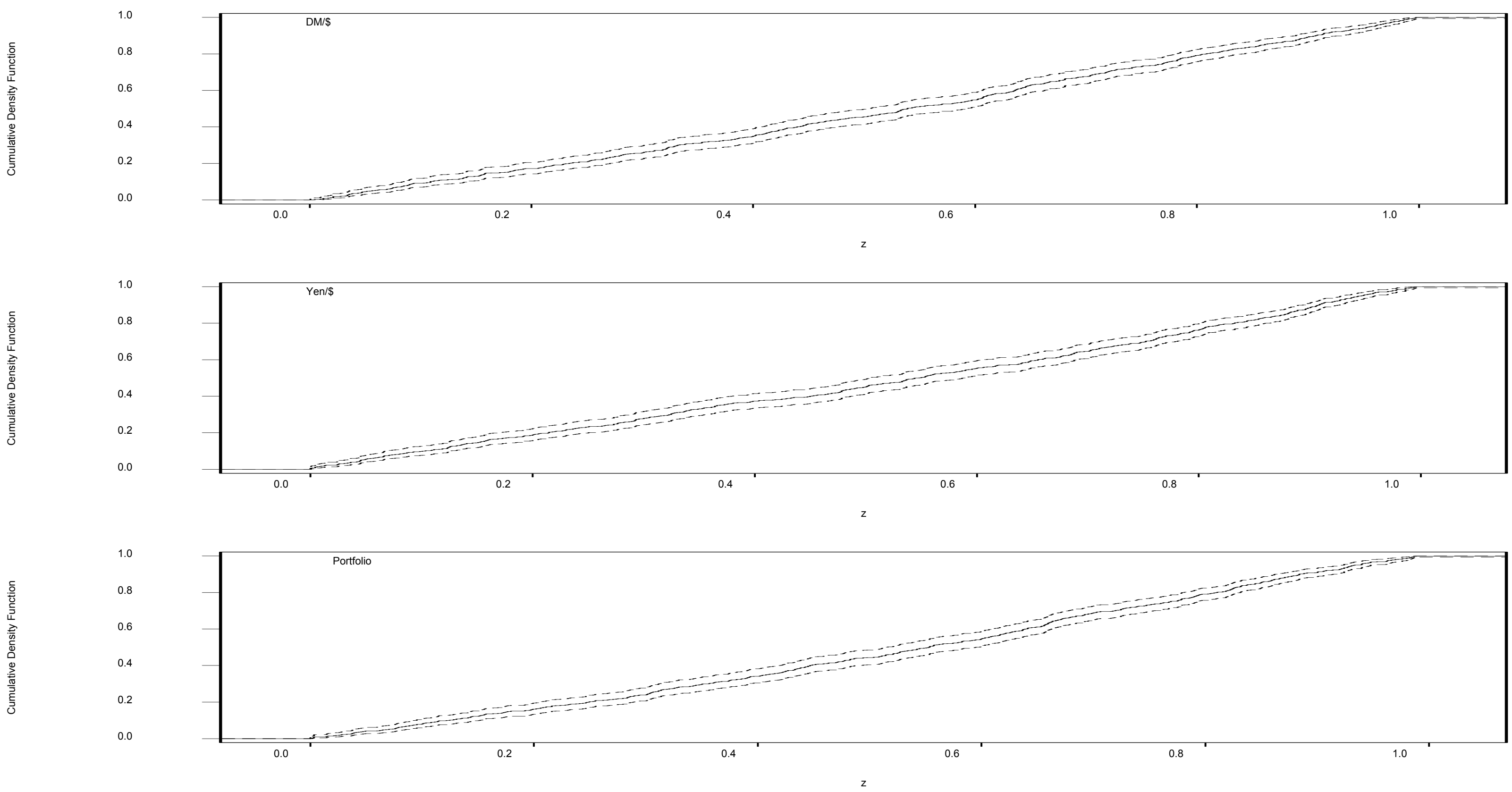
Return Density Forecasts from Lognormal-Normal Mixtures

Recall the lognormal-normal mixture model:

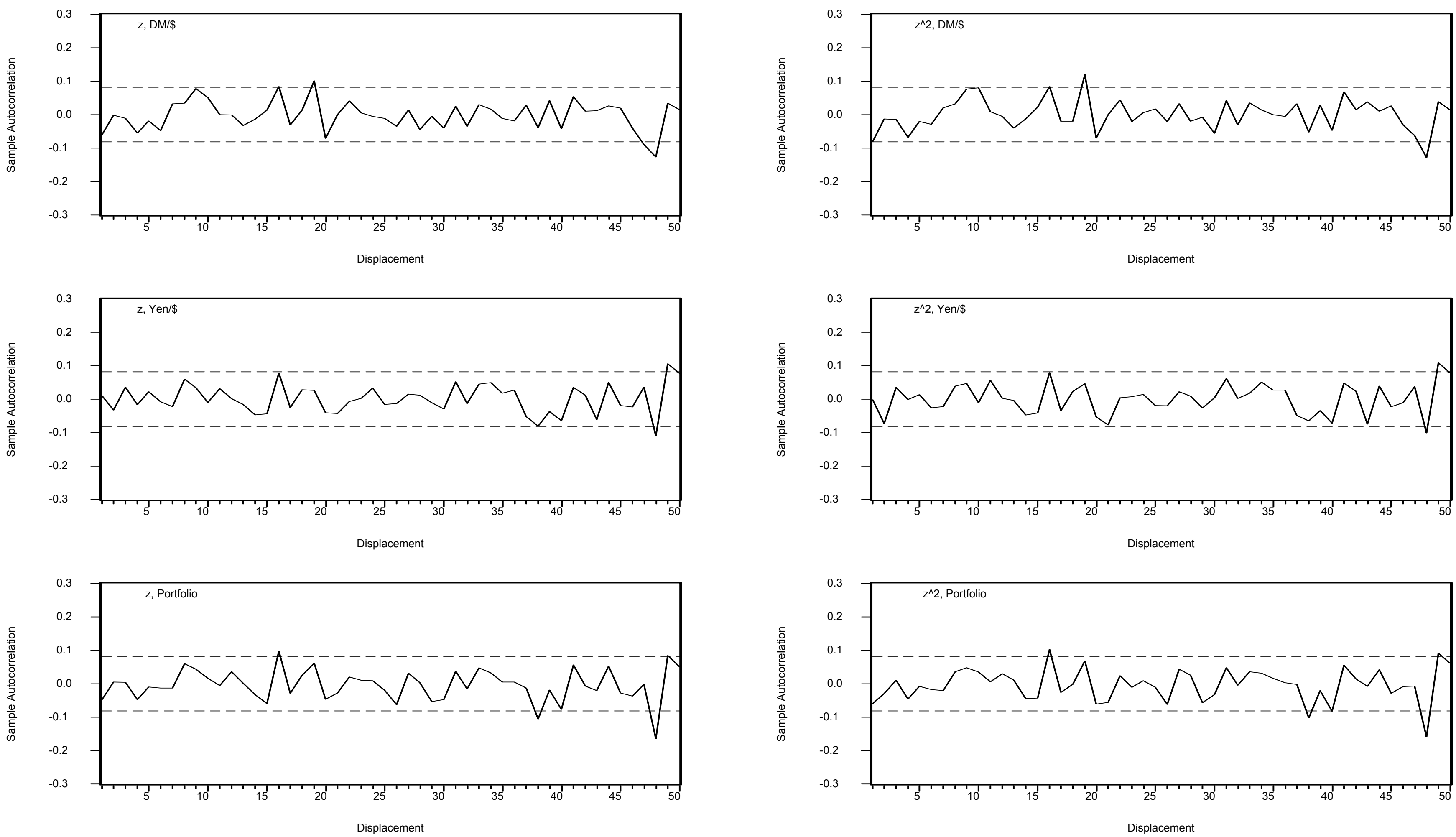
$$r_t \quad = \quad \sigma_t \quad \bullet \quad \varepsilon_t$$

log-normal N(0,1)

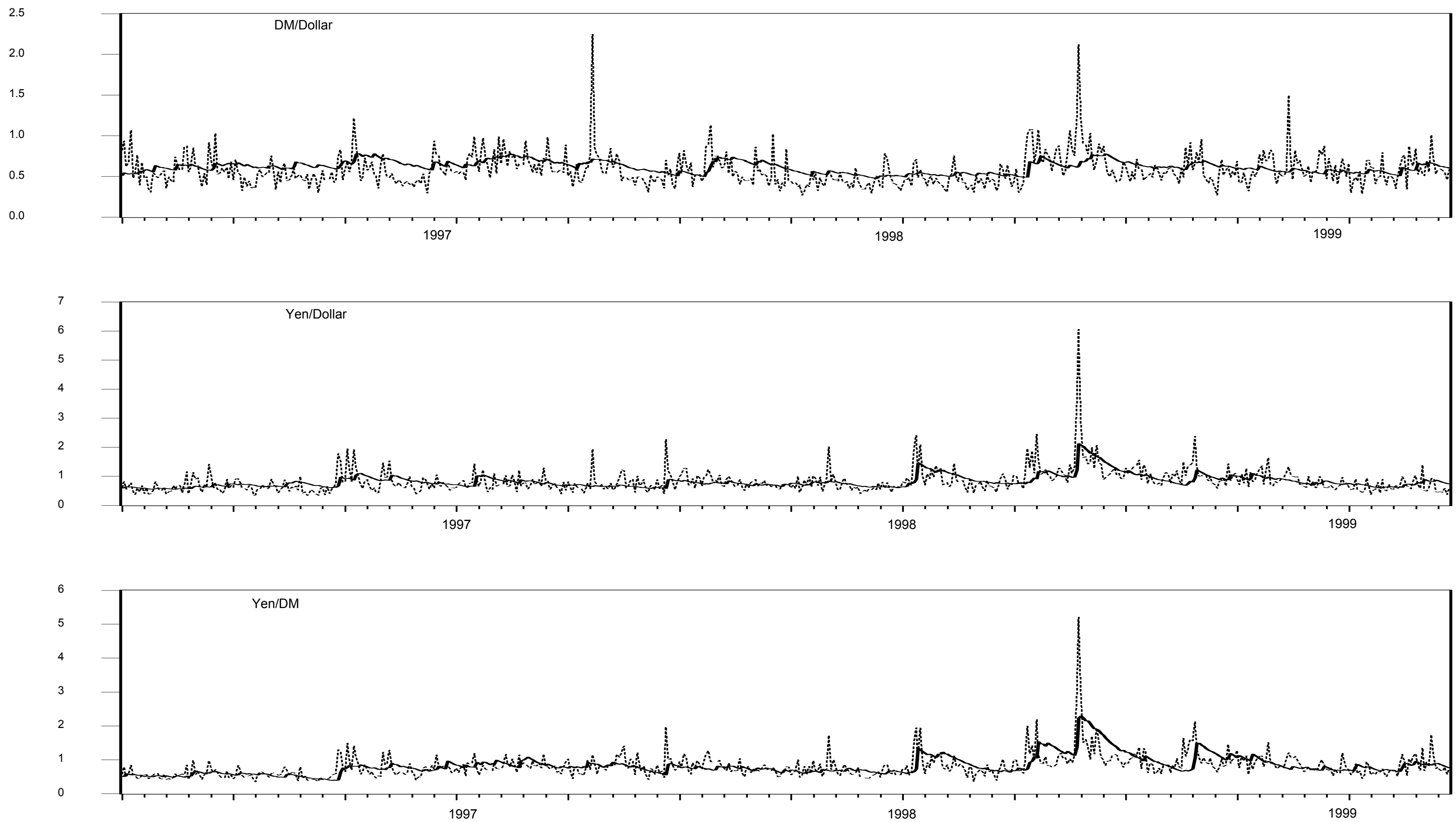
Out-of-Sample One-Day-Ahead Density Forecast Evaluation
CDF of Probability Integral Transform



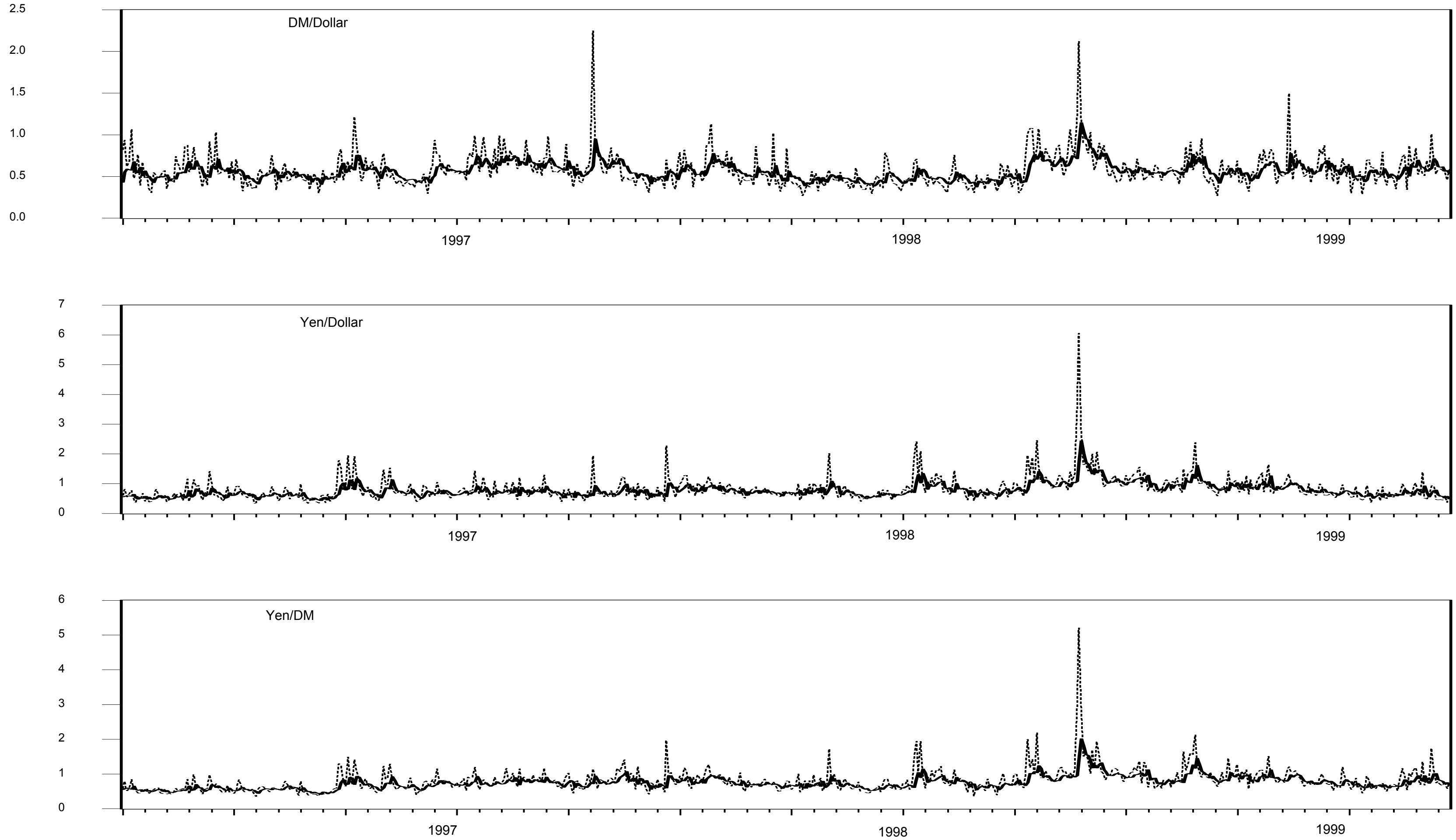
Out-of-Sample One-Day-Ahead Density Forecast Evaluation
Autocorrelations of Probability Integral Transform



Realized Volatility and Out-of-Sample GARCH Forecasts



Realized Volatility and Out-of-Sample VAR-RV Forecasts



The Future

I. Risk Management

Regulatory compliance *and* best practice
Density forecasting, drawdown control, ...

• Microstructure noise: sampling, filtering, ...
"Great Realizations," *Risk Magazine*, 13, 105-108, 2000.

• High-dimensional volatility modeling: factor structure, ...
In progress...

II. Asset Pricing

- Asset pricing: "standard" derivatives...

"Untangling Continuous and Jump Components in Measuring, Modeling, and Forecasting Asset Return Volatility," Working Paper, University of Pennsylvania, 2004.

- Asset pricing: "exotic" derivatives...

"Weather Forecasting for Weather Derivatives," Working paper, University of Pennsylvania, 2004

III. Portfolio Allocation

• Realized beta

"Realized Beta," Working paper, University of Pennsylvania, 2005

• Volatility and market timing

"Financial Asset Returns, Market Timing, and Volatility Dynamics," Working paper, University of Pennsylvania, 2005.

Volatility Timing

$$\min_{w_t} \; w_t' \Sigma_t^{-1} w_t$$

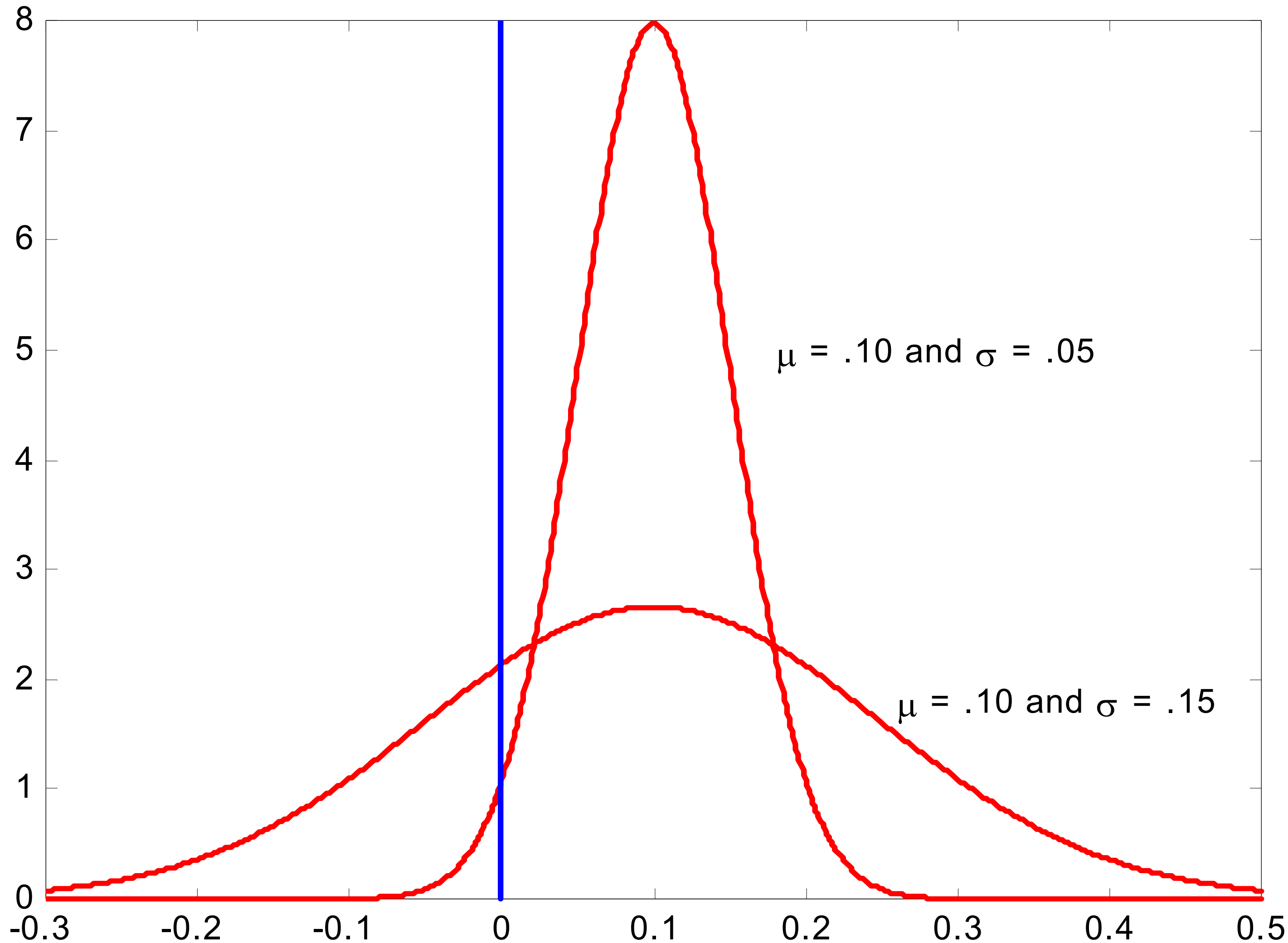
$$\text{s.t.} \quad w_t' \mu + (1 - w_t' 1) R_f = \mu_p$$

$$w_t^* = \frac{(\mu_p - R_f) \Sigma_t^{-1} (\mu - R_f 1)}{(\mu - R_f 1) \Sigma_t^{-1} (\mu - R_f 1)}$$

Fleming et al. (2001, *JF*; 2002, *JFE*):
Utility value of volatility timing: 50 - 200 basis points!

Volatility Timing and *Market* Timing

The Probability of a Positive Return Depends on Volatility



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Volatility as an Asset Class...