



# Do Sophisticated Investors Believe in the Law of Small Numbers?

**Guillermo Baquero**

**Marno Verbeek**

*RSM Erasmus University & Netspar*

[mverbeek@rsm.nl](mailto:mverbeek@rsm.nl)

**Seminar Inquire Europe**

**Oslo, 2 October 2007**

## Introduction and motivation

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- It is well documented that investors in mutual funds and hedge funds chase the winners. Money flows are massively directed to the best performers in the previous year (e.g. Sirri and Tufano, 1998, Agarwal, Daniel Naik, 2003, Baquero and Verbeek, 2006).
- However, there is little evidence that past performance predicts future performance.
- Are investors overreacting to past performance signals?

## The law of small numbers

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Representativeness heuristic: people think that small samples should replicate the true distribution as much as large samples.

Experiments indicate that when subjects are asked to reproduce a truly random series, say tossing a coin, they alternate too often between heads and tails (negative autocorrelation).

This leads to the **gambler's fallacy**. When people observe a streak of signals and are certain the true process is random, they tend to expect a reversal.

## The law of small numbers (2)

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The converse may happen when people think the true process is not random (the coin is unfair, a fund manager is talented,...).

In this case, people may infer that a streak of signals is too long to be random, and expect continuation.

This leads to the so-called **hot-hand fallacy** (first documented by Gilovich et al, 1985, for basketball player's shots).

Momentum strategies in investing are often attributed to this hot-hand bias (overreaction).

## In this paper

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We investigate money flows to and from hedge funds by “sophisticated investors” (institutional investors or high net worth individuals).

By looking at hedge fund investors (rather than retail investors) we hope to separate misconceptions due to lack of experience, or lack of understanding of financial markets, from a psychological bias.

We look at quarterly performance streaks (winner/loser sequences), their impact upon expected future performance, and their impact upon money flows.

## In this paper

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A theoretical model underlying our analysis is provided by Rabin (2002, QJE).

This model, based on Bayesian updating of beliefs, assumes that believers in the law of small numbers, mistakenly think that outcomes are drawn from a small “urn” without replacement, that exactly matches the true distribution. (E.g. an urn with 2 heads and 2 tails for a fair coin.)

The model predicts:

- Agents overinfer the heterogeneity in fund manager quality
- Agents have more extreme beliefs than they should.

## Contributions of this paper

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We model relative performance of a hedge fund from historical performance streaks and other factors.

- The length of the streak is to some extent indicative of future relative performance (consistent with some short-run persistence in performance).

We investigate the response of money flows to the length of the streak, controlling for expected performance and other factors.

- The length of a streak has a significant positive impact on flows, beyond what is justified by expected future performance. The longer the streak, the bigger the impact.

## Investing in hedge funds

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Schematically, we can distinguish three steps:

1. Identifying potentially talented managers, often by their performance track records.
2. Quantitative and qualitative due diligence process.  
*Is performance due to luck/skill? Risk/return characteristics. Incentive mechanisms (fees). Restrictions.*
3. After hiring a manager: ongoing due diligence.  
*Frequent monitoring. Quantitative and qualitative evaluations.*

## Investing in hedge funds and hot hands

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If investors would be skeptical about managerial skill and convinced that the probability of a successful manager is constant (50%), they will be prone to the gambler's fallacy.

As hedge fund investors firmly believe that talented managers exist, they believe in quality dispersion and are prone to overinfer talent from past performance streaks (the hot-hand fallacy).

Further, the longer the streak, the larger the overinference will be. This is precisely what we test in the current paper.

## Data

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We use data from TASS Management Limited on open-end hedge funds reporting in US dollars and excluding funds-of-funds.

Sample contains 752 funds and a total of 7457 fund-period observations from 1994Q4-2000Q1. 249 of them did not report/survive until end of sample period.

*(A future version will update this.)*

We use quarterly data to reduce the impact of return smoothing (due to illiquid investments,..), while allowing reasonable streak lengths in our analysis.

## Money flows

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We consider money flows as representative of the average investor's opinion about a fund.

Money flows are (as usual) derived from the growth rate of a fund's total net assets corrected for the internal growth of a fund due to the return earned.

We consider either %flows or \$flows:

$$\text{CashFlow}_{t+1} = \frac{\text{Assets}_{t+1} - \text{Assets}_t}{\text{Assets}_t} - r_{t+1}$$

$$\text{DollarFlow}_{t+1} = \text{Assets}_{t+1} - \text{Assets}_t (1 + r_{t+1})$$

Panel A : Winner Streaks

Streak Length	Number of observations	Subsequent Liquidation %	Subseq. Persistent Winner %	Subsequent Positive Money Flows (%)	Average Amount of Dollar Flows Invested	Frequency of Wrong Forecasts Up %
1	2818	1.28	48.86	57.38	1618354.31	47.31
2	1319	0.99	52.77	63.76	2143430.16	41.26
3	687	0.44	57.50	70.89	6193009.71	41.07
4	388	0.00	59.79	73.20	8142902.68	37.32
5	224	0.00	62.05	75.89	9715289.70	38.82
6	111	0.00	69.37	77.48	9288168.96	25.58
7	70	0.00	60.00	75.71	8152601.31	35.85
8	41	0.00	60.98	75.61	14411952.51	38.71
9	21	0.00	71.43	76.19	3597137.64	25.00

Panel B : Loser Streaks

Streak Length	Number of observations	Subsequent Liquidation %	Subseq. Persistent Loser %	Subsequent Negative Money Flows (%)	Average Amount of Dollar Flows Invested	Frequency of Wrong Forecasts Down (%)
1	2846	1.76	48.95	44.83	787251.95	47.49
2	1335	2.02	47.72	52.28	-1838814.00	49.71
3	604	6.13	55.96	57.95	-2361213.04	39.71
4	326	8.90	55.21	60.43	-5764902.06	35.03
5	167	10.18	62.28	65.87	-10905250.07	27.27
6	79	11.39	60.76	62.03	-2555103.83	32.65
7	43	13.95	39.53	51.16	-9425391.90	40.91

## Disentangling a rational response from a hot hand bias

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1. We estimate a model explaining future performance (in terms of ranks) from :
  - The length of the streak (persistence)
  - Age, Size, Style, St.Dev of returns, lagged flows
  - Other fund specific characteristics (e.g. fees)
2. Using this model, we obtain forecasts of future performance
3. We estimate a model explaining sign of money flows from :
  - Expected performance
  - The length of the streak
  - Other variables accounting for risk (or preferences)

# Persistence dummies

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We define (2 x) six mutually exclusive dummies:

- $W1 = 1$  if a fund is a winner in previous quarter *only*.  $W1 = 0$  otherwise.
- $W2 = 1$  if a fund is a winner in previous 2 quarters *only*.
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- $W5 = 1$  if a fund is a winner in previous 5 quarters *only*.
- $W6 = 1$  if a fund is a winner in previous 6 quarters *or more*.
  
- $L1 = 1$  if a fund is a loser in previous quarter *only*.  $L1 = 0$  otherwise.
- :
- $L5 = 1$  if a fund is a loser in previous 5 quarters *only*.
- $L6 = 1$  if a fund is a loser in previous  
6 *or more* quarters.

Parameters	OLS estimates including only persistence dummies (A)		OLS estimated, excluding the structure of lagged ranks (B)		OLS estimates including the structure of lagged ranks (C)	
Intercept	0.4938	(69.93)	-0.2036	(-0.84)	-0.2281	(-0.93)
W2	0.0173	(1.40)	0.0092	(0.75)	0.0315	(1.86)
W3	0.0417	(2.61)	0.0255	(1.64)	0.0118	(0.60)
W4	0.0514	(2.60)	0.0250	(1.28)	-0.0116	(-0.51)
W5	0.0879	(3.62)	0.0526	(2.16)	0.0413	(1.53)
W6	0.0845	(4.85)	0.0405	(2.21)	0.0374	(1.72)
L1	-0.0091	(-0.92)	-0.0051	(-0.52)	0.0123	(0.70)
L2	0.0071	(0.58)	0.0190	(1.58)	0.0168	(1.01)
L3	-0.0631	(-3.99)	-0.0384	(-2.41)	-0.0086	(-0.44)
L4	-0.0737	(-3.74)	-0.0453	(-2.31)	0.0060	(0.26)
L5	-0.1070	(-4.26)	-0.0747	(-2.99)	-0.0444	(-1.60)
L6	-0.0860	(-3.51)	-0.0439	(-1.77)	-0.0229	(-0.84)
Rnk lag 1					0.0296	(1.21)
Rnk lag 2					-0.0002	(-0.01)
Rnk lag 3					0.0754	(4.61)
Rnk lag 4					0.0160	(1.09)
Rnk lag 5					-0.0508	(-3.66)
Rnk lag 6					-0.0172	(-1.28)
Control Variables (Size, Age, lagged flows, St Dev, Upside Potential)	NO		YES		YES	
Number of observations	7457		7425		7425	
R <sup>2</sup>	0.0159		0.0521		0.0583	

## This model shows:

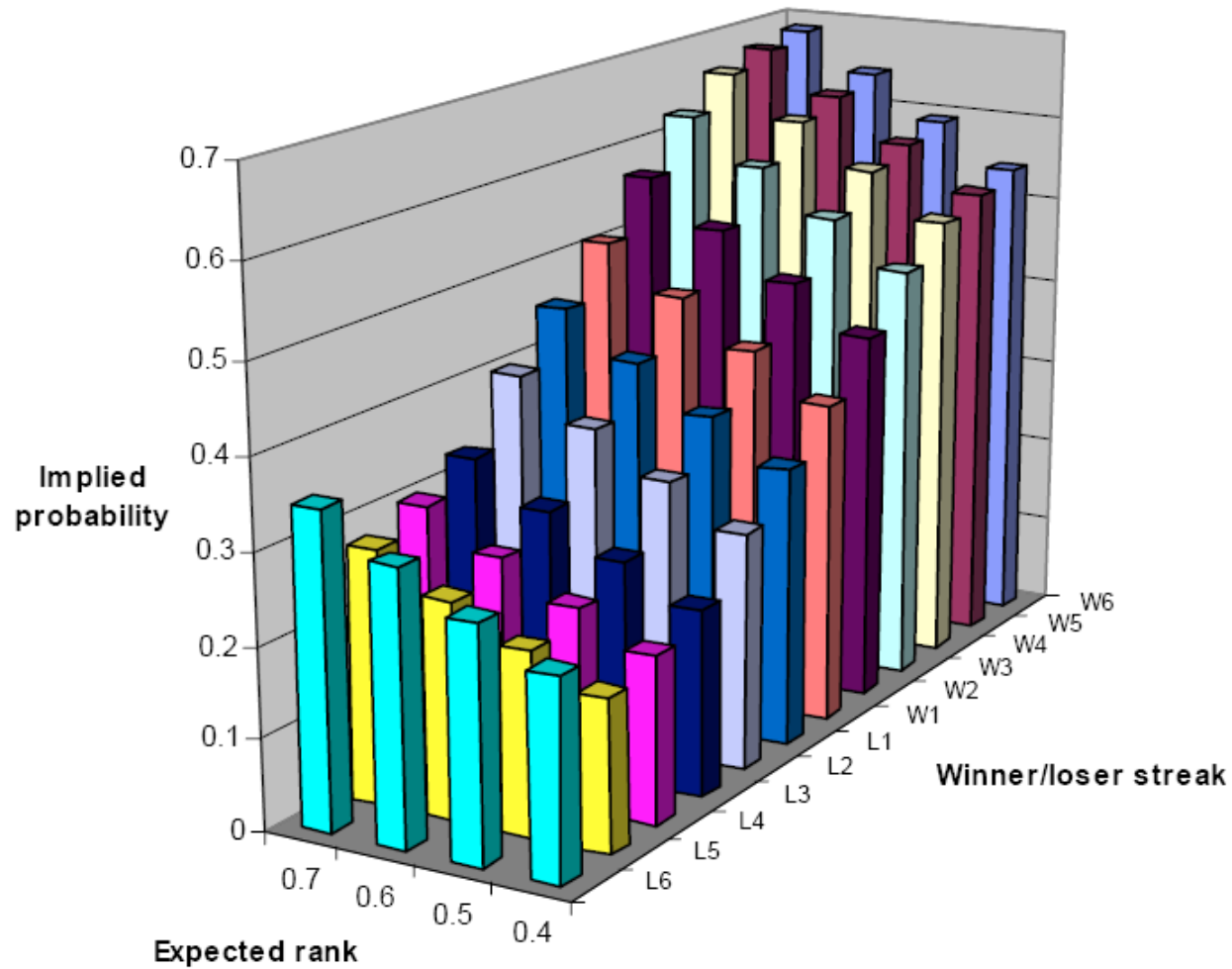
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- Future hedge fund performance is predictable to a very limited extent (about 5-6%), consistent with most empirical studies (e.g. Baquero, ter Horst, Verbeek, 2005).
- Some persistence is present: winners tend to repeat and the longer the streak, the better expected future performance.
- When lagged performance ranks are also included, the picture becomes a bit messy (multicollinearity), but overall there is a significant impact of lagged performance upon future performance.
- We construct forecasts from model C.

Probit model explaining positive  
and negative cash flows.

Parameters	A		B	
Intercept	-0.5493	(-4.34)	0.1341	(0.37)
Expected Rank	1.2044	(4.84)	1.2691	(2.08)
W2	0.1534	(2.87)	0.1473	(2.66)
W3	0.2954	(4.27)	0.2870	(3.97)
W4	0.4450	(4.95)	0.3864	(4.15)
W5	0.4783	(4.13)	0.4265	(3.56)
W6	0.6884	(6.97)	0.4565	(4.30)
L1	-0.0876	(-2.06)	-0.1388	(-3.17)
L2	-0.2453	(-4.70)	-0.2762	(-5.02)
L3	-0.3954	(-5.42)	-0.4652	(-5.93)
L4	-0.4889	(-5.22)	-0.5457	(-5.50)
L5	-0.6354	(-5.01)	-0.6098	(-4.42)
L6	-0.4265	(-3.59)	-0.4194	(-3.35)
Control Variables (Size, Age, lagged flows, St Dev, Upside Potential)	NO		YES	
Number of observations	7195		7195	
Pseudo R <sup>2</sup>	0.0428		0.094	

**Figure 1**  
**Probability of investing implied**  
**by the estimated model of flows (model B, Table V).**



## Predicting the sign of flows

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- Expected performance rank has positive and significant impact. (No evidence of nonlinearity.)
- All dummies indicating the lengths of winning or losing streaks are highly significant.
- The magnitudes of these coefficients is larger for longer streak lengths.
- Thus, money flows are directed (more) towards funds that experience longer winning streaks (and more from those with longer losing streaks).
- Several control variables are statistically significant (lagged cash flows, fund age, risk).



## Predicting the sign of flows

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- Hedge fund investors appear to direct their money flows in response to winning and losing streaks much more than is justified by the expected future performance of the funds.
- This is consistent with the hot-hand fallacy.
- Are investor's allocations really suboptimal?
- Let's consider a simple exercise.

## Consider three investment strategies

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1. **Naïve strategy:** invest in all winning funds (W1...W6);
2. **The average investor's strategy:** invest in all funds with positive money flows
3. **A simple model strategy:** invest in all funds with predicted above median rank (model C).

	Return per qu.	Style adjusted
1	4.46%	0.59%
2	4.59%	0.52%
3	6.31%	1.06%

## Robustness checks (Appendix)

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We have performed a wide range of robustness checks. None of them is able to explain away the strong and clear pattern from Figure 1 (and Table V).

- Explaining growth rates rather than sign of flows: pattern hardly changes (expected rank becomes insignificant).
- Using other thresholds (20<sup>th</sup>, 40<sup>th</sup>, 60<sup>th</sup>, 80<sup>th</sup> percentile) to define winners and losers: similar results.
- Basing rank and persistence on style-adjusted returns rather than raw returns: similar results.

## Robustness checks

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- Including expected performance rank over the next year in addition to the next quarter: expected annual rank is highly significant and negative, but pattern of dummy coefficients remains unchanged.
- Including average performance over the previous year (as a proxy for long-run expected performance?). Is highly significant and positive. The coefficients on the persistence dummies remain significant and increasing, but their combined impact reduces by about 10-30%.

# Conclusions

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- There is some degree of management skill in hedge fund performance. The longer a winning or losing streak, the higher the probability that the streak will persist. In other words, there are hot hands and cold hands.
- The longer a winning (losing) streak, the more likely a fund will have positive (negative) money flow, indicating that the average hedge fund investor follows a trend-following or momentum strategy.

# Conclusions

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- Streaks are not the only predictor of future fund performance. Fund size, age, management fees, incentive fees, managerial ownership and investing style also help predict performance.
- The impact of past performance on flows exceeds that justified by a more robust consideration of other predictive factors, suggesting that investors overestimate the role of management skill in generating returns.
- This is consistent with a “hot-hand” bias driving momentum investing.

## Concluding remarks and discussion

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- Investors do not do better than a naïve strategy of simply investing in past winners.
- Investor's decisions are suboptimal compared to a hypothetical investment strategy based on our simple model forecasting future relative performance.
- Does this really mean these “sophisticated investors” are irrational and believers in the law of small numbers?