Inquire Europe
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Pensions, Bonds, and Inflation

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Two large questions in portfolio analysis

1. Determining probability distributions of future asset returns

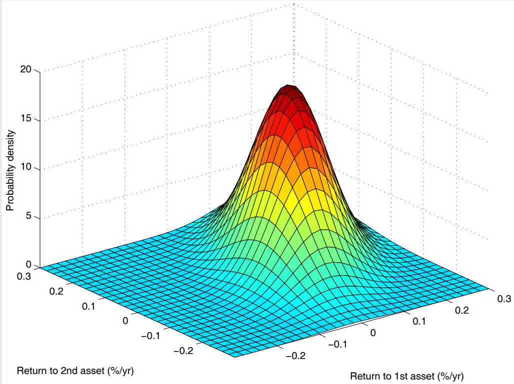
   Depends on asset mix, return horizon, numeraire

2. Calculation of optimal portfolios

   Static (buy and hold), dynamic strategies

   Depends on results of (1), investment horizon, behavior of nontraded asset cash flows, risk preferences

1. Distributions of asset returns



- Standard approach
  1. Describe short-run dynamics of conditional expectations and covariances among returns to various asset classes
  2. Using analytics or simulations, construct longer-run probability distributions implied by short-run dynamics
- To what extent should finance theory be used in implementing this approach?

An atheoretic approach: VARs

- Stack nominal returns to asset classes in R_t
- VAR(p)

$$\begin{pmatrix} R_t \\ \pi_t \end{pmatrix} = \mu + K_1 \begin{pmatrix} R_{t-1} \\ \pi_{t-1} \end{pmatrix} + \cdots + K_p \begin{pmatrix} R_{t-p} \\ \pi_{t-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_{R,t} \\ \varepsilon_{\pi,t} \end{pmatrix}$$

- Conditional expected returns

$$E_t \begin{pmatrix} R_{t+1} \\ \pi_{t+1} \end{pmatrix} = \mu + K_1 \begin{pmatrix} R_t \\ \pi_t \end{pmatrix} + \cdots + K_p \begin{pmatrix} R_{t-p+1} \\ \pi_{t-p+1} \end{pmatrix},$$

$$E_t \begin{pmatrix} R_{t+2} \\ \pi_{t+2} \end{pmatrix} = \mu + K_1 E_t \begin{pmatrix} R_{t+1} \\ \pi_{t+1} \end{pmatrix} + \cdots + K_p \begin{pmatrix} R_{t-p+2} \\ \pi_{t-p+2} \end{pmatrix}$$

- Also model conditional covariances of shocks $\varepsilon_{R,t}, \varepsilon_{\pi,t}$. Some possibilities:
  1. Multivariate normal shocks
  2. Bootstrap shocks
  3. Use multivariate GARCH model
- Estimate parameters with observed inflation and return data
- Resulting model is simulated to generate distribution of returns at any desired horizon

- Two main problems with atheoretic VAR

  1. Specification and estimation

     Many possible specifications, many parameters to estimate

     Steehouwer's paper details problems and possible solutions

  2. Conceptual – is there information not in the VAR that is useful for forecasting?

- *Informal* use of finance theory can help

VAR mixed with some finance

- Example: Treasury bonds

  Investor expectations of future bond returns are impounded into current prices (yields)

- Therefore use yields instead of returns in a VAR(1)

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \mu + K \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{pmatrix}$$

- Estimate model, simulate yields and inflation, then construct simulated returns

A more formal finance approach

- Big finance idea: Equal compensation for equal risk

  Expected excess returns are proportional to exposure to priced risks

- Sketch of approach

  1. Choose economic model of expected excess returns (e.g., CAPM)
  2. Choose model for dynamics of aggregate risk compensation

     Either atheoretic or one from economic theory
  3. Choose model for dynamics of conditional variances, covariances

     Plays two roles: affects expected excess returns and used in generating simulations
  4. For nominal and/or real returns, add models for dynamics of short-term interest rate, inflation

What can go wrong?

- Everything. In particular,
  - Poor model of priced risks
    - Aggregate market risk? (CAPM)
    - Above + nominal exchange rate risk?
    - Above + inflation risk?

    Paper by Moerman and van Dijk contrasts the latter two possibilities
  - Poor description of variations in risk compensation

    Example–what determines variations in the long-bond premium over time?
- Extreme example of results

  Duffee (J of Finance, 2002) shows that existing models of the term structure perform worse than the most naive alternative

So why use a formal economic model?

- We’re getting better at building them
- Accuracy of atheoretic models constrained by data limitations
- Portfolio allocation decisions often require one

Formally modeling the term structure

- Term structure linked to macroeconomy

  Short-term default-free nominal rate, inflation, GDP growth follow VAR process

$$\begin{pmatrix} r_t \\ \pi_t \\ \Delta g_t \end{pmatrix} = \mu + K \begin{pmatrix} r_{t-1} \\ \pi_{t-1} \\ \Delta g_{t-1} \end{pmatrix} + \Sigma \begin{pmatrix} \varepsilon_{r,t} \\ \varepsilon_{\pi,t} \\ \varepsilon_{g,t} \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_{r,t} \\ \varepsilon_{\pi,t} \\ \varepsilon_{g,t} \end{pmatrix} \sim MVN(0, I)$$

- Model completed by adding time-varying compensation investors require to face these risks

$$\text{expected excess return per unit of risk } i = \lambda_{0i} + \lambda_{1i} r_t + \lambda_{2i} \pi_t + \lambda_{3i} \Delta g_t$$

Ang, Dong, Piazzesi, ``No-arbitrage Taylor rules,'' 2005 working paper, University of Chicago GSB

Why use this model?

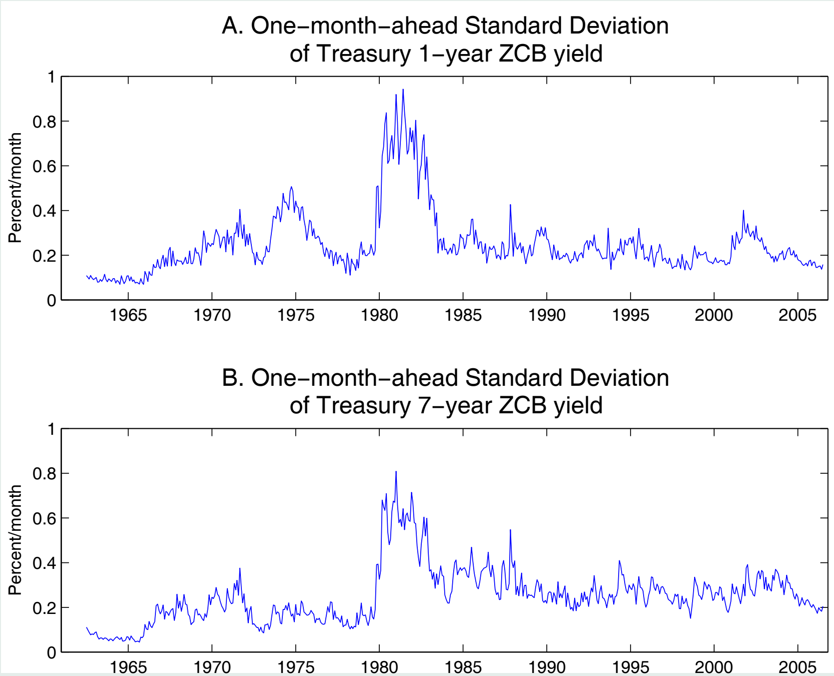
- Used to study relation between term structure and macroeconomy

  What does shape of term structure tell us about future inflation, output growth?

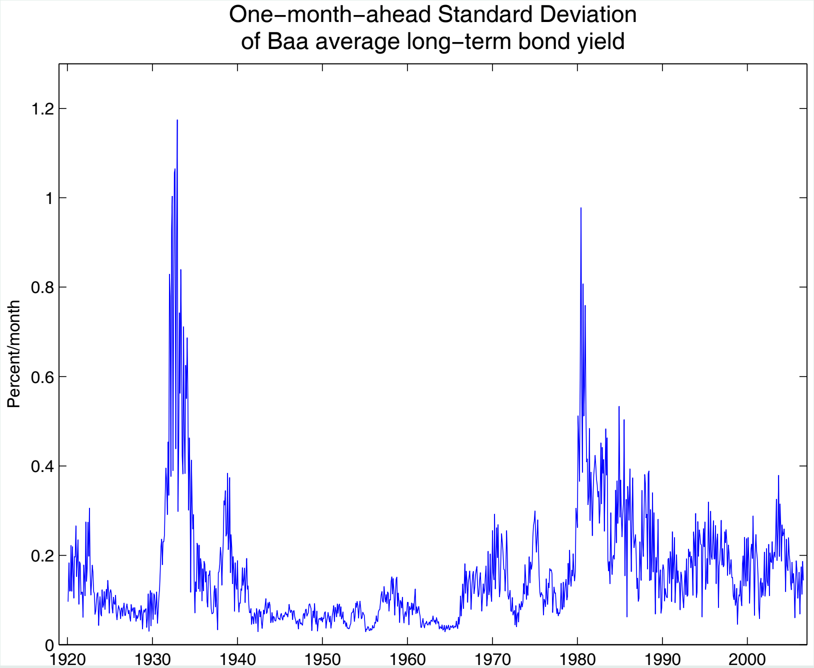
- Better at forecasting future bond returns than some atheoretic models
- Can easily be expanded to include other macro or bond factors

Why not use this model?

- Assumes normally-distributed shocks, constant variances over time
- Assumption overwhelmingly violated in the data
- Implication is that model is poor choice for generating distributions of future yields, bond returns



- Related note – Gaussian assumption even worse for corporate bond yield behavior



- Why not add time-varying interest rate volatility to the model?

  1. Math is much harder

     Only way we know how to do it (affine term structure models) results in less cross-correlation flexibility

  2. Messes up empirical fit of bond risk premia

     Finance theory:

     expected excess returns = $\sum$ (price of risk) $\times$ (amount of risk)

     But hard to find relation between volatilities and expected excess returns in the data.

Regression forecasts of Treasury returns

- Forecasting variables are slope of term structure (5yr ZCB yield - 3mon Treasury yield) and conditional standard deviation of 7yr ZCB yield)
- Forecasted return is monthly return to portfolio of Treasury securities with maturities between 5 and 10 years, less one-month T-bill return
- Samples are July 1962–Dec 2005 and Jan 1985–Dec 2005

Sample	Forecasting Variable	Estimated Coefficient	Robust t-statistic	P-val	R^2
1962–2005	Volatility	1.344	1.39	0.16	0.009
1962–2005	Slope	0.269	2.50	0.01	0.028
1985–2005	Volatility	0.346	0.20	0.84	0.000
1985–2005	Slope	0.222	2.08	0.04	0.017

$$\text{expected excess returns} = \sum (\text{price of risk}) \times (\text{amount of risk})$$

- Implication of this evidence

  Price of risk varies substantially over time

  Part of this variation is inversely correlated with interest rate volatility

- This is very hard to formally model

  Easiest to pretend volatility does not change, while price of risk varies

- Assumption works for fitting observed time-variation in expected excess returns

  But does not work for modeling time-variation in volatilities, and thus in modeling distributions of returns at long horizons

What drives variations in bond risk premiums?

- In model, fundamental risks are shocks to inflation, short rate, output growth

  (arbitrary decomposition)

- In model, price per risk varies *only* with levels of same variables

  – Not based on economic theory, but on parsimony

    Economic theory: long-run behavior of macroeconomy (e.g., five years of booming stock market means that risk-tolerant investors are relatively more important)

    But empirically, little support for such economic theories in bond market

Some preliminary evidence

- Continue to model "fundamentals" as inflation, output growth, short rate

  VAR(1)

- But let risk premium for each shock depend on both levels of fundamentals *and* some other, hidden variable

- Data say that level of fundamentals are unimportant in driving variations in risk premia

  A nihilistic view of the world . . .

``Are variations in term premia related to the macroeconomy?'' working paper, http://faculty.haas.berkeley.edu/duffee

Are fundamentals irrelevant?

- Earlier discussion indicates macro variables unimportant in explaining variations in risk premia

  But can still be important for determining future path of short-term interest rates

- If macro variables matter for explaining interest rates, then when macro variables are highly volatile, interest rates should be too

Standard deviations by decade

Quarterly inflation from U.S. GDP price deflator, output growth is real U.S. GDP growth. All variables in percent/year.

	47–55	56–65	66–75	76–85	86–95	96–05
Output growth	5.50	4.69	3.98	4.56	1.98	2.00
Inflation	3.45	1.07	2.34	2.36	0.91	0.81
3-month T-bill yield	0.47	0.79	1.29	3.05	1.73	1.77
5-year ZCB yield	–	0.52	1.06	2.36	1.22	1.27

Distributions of asset returns

- Eventually, rigorous finance theory should help improve accuracy of probability distributions of future asset returns
- But state of the art no-arbitrage models are not well-designed for this purpose

  Approaches that are less theoretically motivated are probably more useful at present

Calculation of optimal portfolios

1. 1950s academic advice: mean-variance efficiency

   Maximize expected return per unit of standard deviation

2. 1970s academic advice: also hedge variations in future investment opportunities

   Reinterprets (1) as myopic investment strategy

Unlike (1), calculation of optimal portfolio that accounts for (2) is extremely difficult math problem

Two approaches – calculate optimal portfolios in simplistic settings or approximate, “pretty good” portfolios in more realistic settings

Ingredients of optimal portfolio calculation

1. Dynamic model of investment opportunities

   expected returns to a set of assets, return standard deviations, correlations among returns

2. Model of investor utility

   Needed to evaluate tradeoffs between myopic strategy (maximize Sharpe ratio) and strategy of hedging future investment opportunities

   Specifies risk aversion, horizon, nontradeable risky assets (e.g., human capital)

3. Specification of any trading restrictions

   Short sales allowed? Trading costs?

Example: Time-varying short-term real rate

- Rate varies predictably over time

  Affects expected real and nominal returns to stocks, bonds

- Risk premia, volatilities assumed constant – allows for exact calculation of optimal portfolio

- Main intuition

  - Myopic strategy is to invest heavily in bonds
  - But long-horizon investors should have significant investment in long-term inflation-indexed bonds.

    If no such bonds, nominal long-term bonds help, but create exposure to inflation risk

- Papers by Nijman and de Jong study this setting in detail

Alternative approaches

- Paper by Buraschi
  - Correlations between asset returns vary through time
  - Leads to time-varying Sharpe ratios that should be hedged
- More general variations in expected returns to assets

  `Campbell, Chan, Viciera, ``A multivariate model of strategic asset allocation,'' Quarterly Journal of Economics 2003`

  - VAR describes dynamics of expected returns
  - Approximate investment strategy can be computed

Should we trust this advice?

- Logic of hedging future investment opportunities is solid
- But practical importance of hedging depends on magnitude of predictable fluctuations

  Issues here are identical to those in constructing probability distributions of future returns – how do conditional means and covariances of returns vary over time?

  But implications of optimal portfolios are even more sensitive to these dynamics than are distributions of future returns

- Optimal portfolio critically depends on
  1. Sources of largest variations in future investment opportunities
  2. Individual preferences for avoiding risk and smoothing consumption
- But little is known about (1)

  Because of level of difficulty, academic work focuses on settings that are tractable, not necessarily most important

  In particular, is it more important understand how covariances vary or expected returns? We do not know.
- Models of (2) seem plausible ...

  ... but if typical investors had the preferences our models assume, the variations in future investment opportunities that are specified in the models could not occur

A concluding cautionary note

- Academic research is similar to thinking out loud

  Start with an idea, see where it leads

- To better understand an issue, good research often ignores everything else

  Critical issues about which the researcher has nothing to say are often unmentioned

  Big example – importance of time-variation in conditional volatility for determining distributions of future excess returns and future investment opportunities

- Thus academic research can be misleading to those who are not actively engaged in it (even to other academics!)