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# **Pension Fund Investments and the Valuation of Liabilities under Conditional Indexation**

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## Motivation

*"Even if only nominal bonds are available, conservative long-term investors should hold large positions in long-term bonds if they believe that inflation risk is low, as we have estimated it to be in the USA in the period 1983-99. In this sense, the message of this chapter might be summarized as 'Bonds, James, Bonds'. Inflation risk is however a serious caveat."*

John Campbell and Luis Viceira (2002), *Strategic Asset Allocation*, p.87

## Key questions of the paper

- Investigate role of bonds in portfolio of long-term investors (such as pension funds)
  - Campbell-Viceira quote suggests substantial investments in long-term bonds
  - how to deal with inflation risks?
  - emphasize the role of Index Linked Bonds
- Replication of Index Linked Bonds if there is no market for ILBs
- How does inflation risk affect the value of pension liabilities?
  - conditional indexation rules
  - interaction with investment policy

## Main results

- Optimal investment portfolio is a mix of speculative (mean-variance efficient) portfolio and a hedge portfolio against long-term risks (reinvestment risk and inflation risk)
  - Long-term Index Linked Bond is the ideal hedge instrument
- In the absence of long-term ILB's *medium*-term nominal bonds are the best substitute
  - strike the right balance between reinvestment risk hedge and inflation risk hedge
- Conditional indexation clauses can have quite high value
  - value depends on the investment portfolio

# Modern Portfolio Theory

- Mean variance optimal portfolios
  - lowest possible risk (return variance) for given expected return
- With historical return data, almost no investment in nominal bonds
  - bond return is very low compared to risk
- How much to invest in stocks depends on risk aversion
- Traditional advice to individual investors
  - young have longer horizon, so are less risk averse high fraction of equity investments
  - old should save everything risk free (cash)
- Criticism (Bodie): lower risk aversion for young is poorly motivated

## Long-term investment theory

- Investment policy designed to support consumption now and in the future
- Analysis pioneered by Samuelson and Merton and recently extended by Campbell and Viceira, Brennan and Xia etc.
- optimal portfolio consists of a speculative part and a hedge against changes in investment opportunities
  - e.g. interest rate risk and inflation risk

## The Brennan and Xia (2002) model

Risk factors and state variables are

1. stock prices,  $S$
2. real interest rate,  $r$
3. expected inflation,  $\pi^e$
4. unexpected inflation / price level,  $\Pi$

Model structure:

$$dS/S = \mu_S dt + \sigma_S dZ_S$$

$$dr = \kappa(\bar{r} - r)dt + \sigma_r dZ_r$$

$$d\pi^e = \alpha(\bar{\pi} - \pi^e)dt + \sigma_\pi dZ_\pi$$

$$d\Pi/\Pi = \pi^e dt + \sigma_\Pi dZ_\Pi$$

## Objective function

The pension fund maximizes CRRA utility of real wealth at investment horizon  $T$

$$\max \mathbf{E} [U (W_T/\Pi_T)], \quad U(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

subject to nominal wealth dynamics

$$dW/W = [x'\mu + (1 - \iota'x)R_f]dt + x'\sigma dZ$$

- asset returns:  $\mu$  expected return,  $\sigma$  volatility,  $\rho$  correlation matrix
- $R_f$  is the nominally risk free rate
- $x$  are the portfolio weights

## Optimal portfolio (1)

Optimal portfolio weights are given by

$$x^{opt} = \frac{1}{\gamma} x_{MVE} + \left(1 - \frac{1}{\gamma}\right) x_{HEDGE}$$

- Split between speculative and hedge portfolio
  - weights depend on investors degree of risk aversion
- $x_{MVE}$  is the mean-variance efficient portfolio
  - maximizes expected growth rate of investment value
  - determined by Sharpe ratios and correlation structure of risky assets ( $x_{MVE} = \Sigma^{-1} \sigma \lambda$ )
  - stocks will be significant part of this portfolio

## Optimal portfolio (2)

- $x_{HEDGE}$  is the optimal hedge against long-term risks
  - portfolio weights can be found as the slope coefficients in a regression of the return on an index linked bond with maturity  $T$  on the vector of returns on all available assets

$$R_{ILB} = \alpha + \beta' R + \epsilon$$

- portfolio with 'best' replication of ILB return
- suggests an important role for Index Linked Bonds

## Calibration

- Parameter values from Brennan-Xia study
- Three cases, with nominal and indexed bonds
- Impose borrowing constraint ( $x_{cash} \geq 0$ )

## Optimal portfolio: numerical results

<i>maturity</i>	stock	bond	cash	stock	bond	cash	stock	ILB	cash
	<i>20yr</i>			<i>5yr</i>			<i>20yr</i>		
risk premium	3.16	2.17		3.16	0.83		3.16	1.09	
st. dev.	15.80	23.61		15.80	8.03		15.80	10.94	
correlation		0.08			0.10			0.13	
speculative	1.23	0.32	-0.55	1.21	1.05	-1.26	1.21	0.68	-0.89
hedge	0.07	0.18	0.75	0.05	0.78	0.17	0.00	1.00	0.00
$R^2_{hedge}$			<b>0.17</b>			<b>0.34</b>			<b>1.00</b>
investor type:									
aggressive	0.84	0.16	0	0.99	0.01	0	0.94	0.06	0
moderate	0.65	0.25	0.10	0.53	0.47	0	0.47	0.53	0
conservative	0.30	0.21	0.49	0.26	0.74	0.03	0.19	0.81	0

## Optimal portfolio: insights

- Speculative portfolio contains large fraction of stocks
  - and has substantial leverage
- Hedge portfolio contains (almost) no stocks
- For reasonable levels of risk aversion borrowing constraints are almost always binding
- Long-term nominal bonds have very low hedge effectiveness
  - too sensitive to expected inflation risk, hence very volatile
  - best nominal bond maturity is around 5 years
- Index linked bonds provide best hedge effectiveness
  - hedge perfectly against real interest rate risk and inflation risk
  - even medium term ILB's are quite effective hedge instruments

## Human capital

The model can be extended to include human capital

- present value of wage income during working life
- total wealth equals human capital plus financial wealth

Portfolio allocation decision depends on ratio of human wealth to financial wealth

- for young workers, human capital is large fraction of total wealth
- allows to leverage up the speculative portfolio part
- human capital also serves as a hedge against long-term risks
  - such as real interest rate and fluctuations in price or wage level

## Optimal portfolio with human capital

If wages are perfectly indexed to inflation, human capital returns are perfectly correlated with indexed bond returns.

For that case the optimal portfolio weights are

$$x^{opt} = \frac{1}{\gamma_h} x_{MVE} + \left(1 - \frac{1}{\gamma_h}\right) x_{HEDGE}$$

- $\gamma_h = \gamma(1 - h)$  is the effective risk aversion, where  $h$  is the ratio of human capital to total wealth
- $x_{MVE}$  is the same mean-variance efficient portfolio as before
- $x_{HEDGE}$  is the optimal hedge against long-term risks, corrected for automatic hedge provided by human capital
  - shorter maturity ILB best hedge instrument

## Optimal portfolio with human capital

	stock	nominal bond	stock	ILB
moderate				
$h = 0.0$	0.53	0.47	0.47	0.53
$h = 0.5$	0.96	0.04	0.86	0.14
$h = 0.8$	2.24	-1.24	2.04	-1.04
conservative				
$h = 0.0$	0.26	0.74	0.19	0.81
$h = 0.5$	0.42	0.58	0.30	0.70
$h = 0.8$	0.88	0.12	0.63	0.37

## Valuation of pension liabilities

- In Defined Benefit plans, cash flows are often indexed to price or wage level
- In many UK contracts, this indexation is limited
- In the Netherlands, indexation is typically conditional on the financial situation of the fund
  - value will be dependent on investment policy!
- Contingent claim (option pricing) methods needed for valuation
- Model structure is useful for this valuation exercise

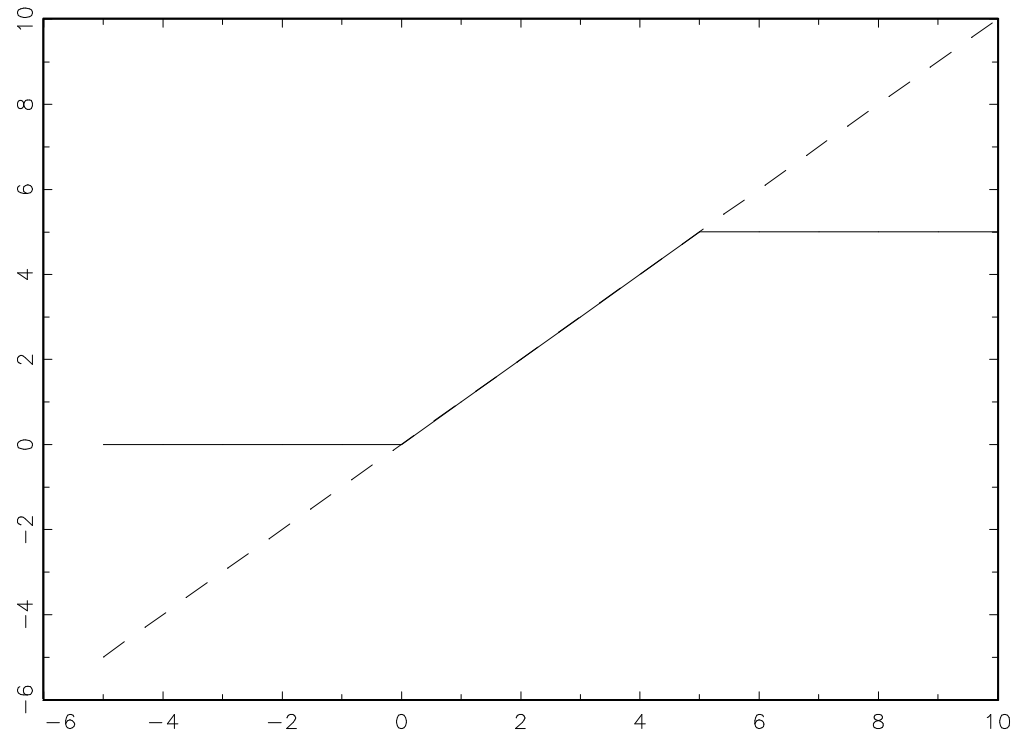


Figure 1: Collar type indexation rule

## Contingent claims valuation

A nominal payoff  $X_T$  has time 0 value

$$X_0 = E[M_T X_T]$$

where  $M_T$  is the Stochastic Discount Factor (a.k.a. deflator or pricing kernel) implied by the Brennan-Xia model. It takes the form

$$M_T = \exp \left\{ - \int_0^T R_f(s) ds \right\} \exp \left\{ -\frac{1}{2} \psi' \rho \psi T + \int_0^T \psi' dZ \right\}$$

The first part is the usual risk-free discount factor

The second part a stochastic part that is high when asset values are low

- implies that options that pay off in 'bad' states have high prices
- $\psi = -\rho^{-1} \lambda$  is determined by the Sharpe ratio's of traded assets

## Valuation of DB pension

- Defined Benefit pension with full indexation

$$L_0 = LE[M_T \Pi_T]$$

- Determined by price of index linked bond paying off at time  $T$ 
  - this ILB is the perfect hedge instrument for  $L$

## Limited indexation

- Suppose indexation is limited to a maximum of 5% per year on average
- Payoff of this pension is  $L \min\{\Pi_T, 1.05^T\}$ , or

$$X_T = L\Pi_T - L \max\{\Pi_T - 1.05^T, 0\}$$

unconditional DB plus a written call option on inflation

- In practice, limit applies every year: path dependent option

$$X_T = L \prod_{t=1}^T \min\{\Pi_t/\Pi_{t-1}, 1.05\}$$

- Numerical valuation of this option by simulating paths for the pricing kernel and the option payoff

## Funding ratio

- In Netherlands, decision to grant indexation depends on the funding ratio, defined as ratio of assets to liabilities

$$FR = A/L$$

- If FR falls below threshold level (e.g. 100%) indexation is limited or skipped altogether
  - contracts are not well specified, but this is observed policy
- Option structure: call option on inflation triggered by FR

$$X_T = L \prod_{t=1}^T \max\{\Pi_t/\Pi_{t-1} I(FR_t > 1), 1\}$$

- The value of this pension depends on the investment policy!

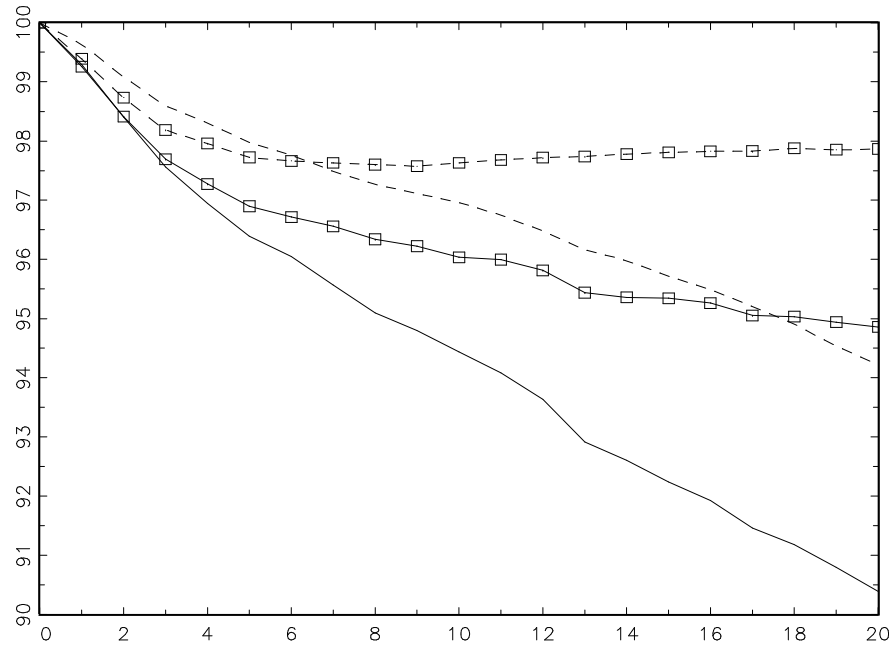


Figure 2: Present value of pension with conditional indexation

Straight lines: moderate investment policy; dashed lines: conservative investment policy. Lines with squares are with ILBs in the portfolio.

## Conclusions

- Bonds provide hedge against long term risks
  - Index Linked Bonds are ideal hedge instrument
  - *medium*-term nominal bonds are the best substitute
- Speculative investments, e.g. stocks, depends on risk attitude
  - asset allocation can be more speculative for young investors with much human capital
- Conditional indexation clauses can have quite high value
  - value depends on the investment portfolio
- Extensions / work in progress:
  - asset allocation decision for home owners (with Joost Driessen and Otto van Hemert)
  - valuation of pension liabilities with unhedgeable wage indexation risk