Hedging Currency Risk: a Regret-Theoretic Approach

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Abstract

Contrary to the predictions of existing normative currency-hedging models, a wide diversity of hedging policies is observed among institutional investors. We propose an alternative model of optimal currency-hedging choices based on regret theory, a normative and axiomatic behavioral theory. With hindsight, investors may experience regret of not having taken the *ex post* optimal hedging decision; i.e. full hedging if the foreign currency depreciated, or no hedging if the foreign currency appreciated. Hence, investors include expected future regret in their objective function. As a result, our model features two components of risk: traditional risk and regret. We derive closed-form optimal hedging rules using the Arrow-Pratt approach and highlight the difference with the traditional expected-utility results. We find that differences in the level of regret aversion among investors may explain why we observe such a wide diversity of currency hedging policies among institutional investors.

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1. Introduction

"I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn't in it--or if it went way down and I was completely in it. My intention was to minimize my future regret, so I split my [pension scheme] contributions 50/50 between bonds and equities."
Harry Markowitz.[1]

Portfolio investors have progressively accepted the argument that international diversification provides risk/return benefits[2]. However, the currency dimension has remained an emotional issue and currency hedging is a sensitive decision. Attractive local-currency returns on foreign asset market can be swamped by a depreciation of the foreign currency. Conversely, the return on foreign currency can provide a major portion of the total return of international investments when the domestic currency is weak. The currency hedging decision is a simple one: what currency hedge ratio (proportion of foreign asset value hedged against currency risk) should be adopted. In other words, should international assets be fully hedged against currency risk, not hedged, or partially hedged?

The currency hedging policies adopted by investors seem quite diverse. Figure 1 gives the distribution of currency-hedging benchmarks for institutional investors delegating the currency hedging decision to overlay managers.[3] Data comes from a survey (Harris-2004) conducted in 2004 by Mellon/Russell and covers 563 accounts of institutional investors worldwide. Looking at the worldwide average, 39% of investors adopt a no-hedging policy, 34% of investors adopt a 50% hedging policy, 14% of investors adopt a 100% hedging policy and 13% of investors adopt some other hedge ratio. Because these numbers reflect long-term policy benchmarks, they cannot be explained by short-term expectations on currency movements, but primarily by risk considerations or some behavioral attitude. The wide diversity in hedge ratios is puzzling. One objective of this article is to provide an explanation for the observed diversity in hedging policies whereas existing normative models support uniform policies across investors.

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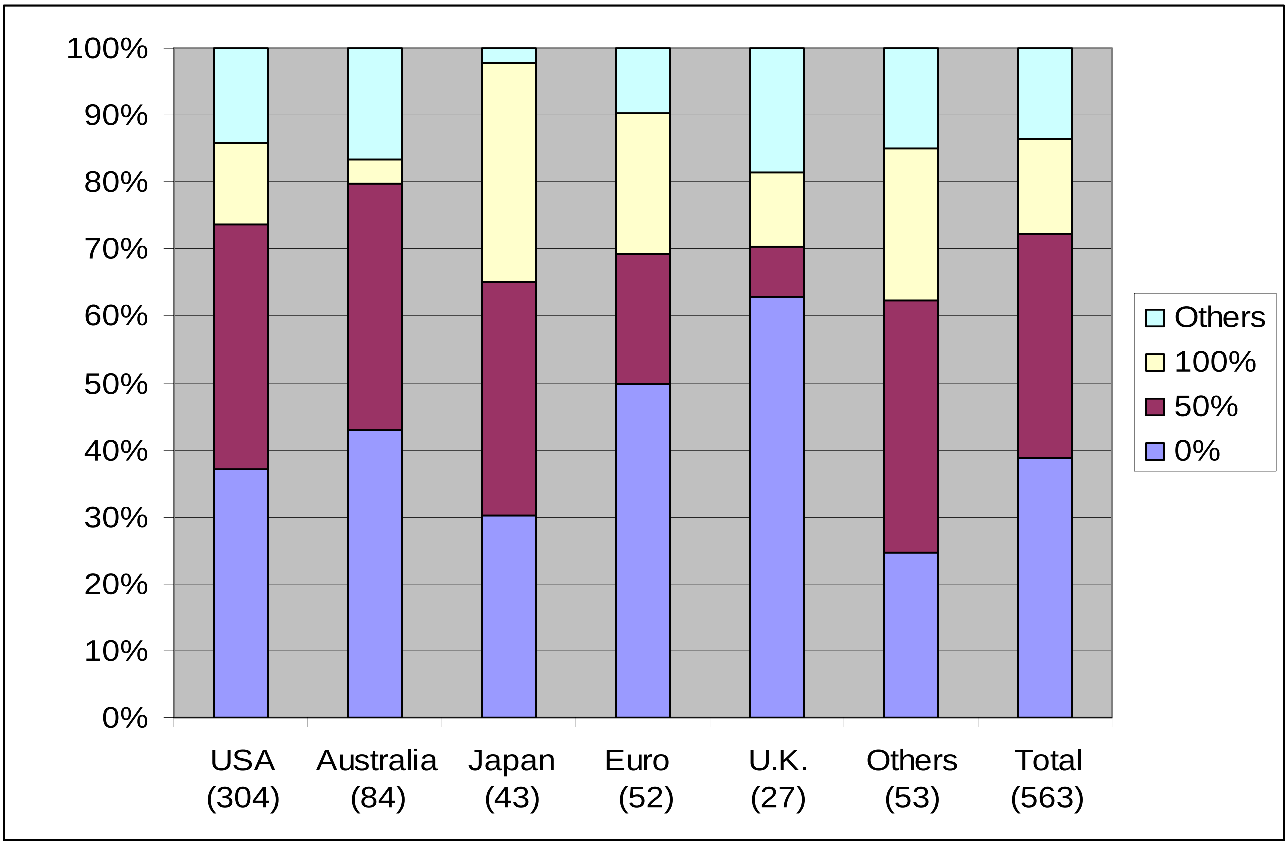
[1] As quoted in Jason Zweig, "How the Big Brains Invest at TIAA-CREF", *Money*, 27(1), p114, January 1998.

[2] In this paper we will use US investors as domestic investors.

[3] The benchmark currency hedging policy drives the hedge ratio adopted by the currency overlay manager who can deviate based on tactical currency expectations and risk assessment. The fact that the data reported in Figure 1 represent benchmarks assigned to currency overlay managers, not the actual hedge ratios, can introduce some biases.

Figure 1: Distribution of hedge ratio for investors from several base currencies

The figure gives the distribution of currency-hedging benchmarks for institutional investors delegating the currency hedging decision to overlay managers. Data comes from a survey (Harris-2004) conducted in 2004 by Mellon/Russell and covers 563 accounts of institutional investors worldwide. Each column gives the distribution of the hedge ratio for investors from a given base currency, as well as the number of accounts in that base currency. The last column gives the distribution of all accounts.



Traditional finance relies on expected utility maximization and uses the expected return/risk paradigm to search for an answer. Some researchers have developed global market equilibrium models to derive optimal currency hedging rules. Solnik (1974) and Adler and Dumas (1983) derive an international asset pricing model where investors from different countries use their own currency as numeraire. Therefore investors from different countries view asset expected returns and risks differently because of foreign exchange uncertainty. Because currency movements reflect much more than adjustment to inflation differentials (purchasing power parity), currency risk is a real risk. Global equilibrium models conclude that all investors should hold a combination of their own risk-free asset (risk-free in home currency) and the world market portfolio partly hedged against currency risks. Hence the risky portfolio is identical for all investors and made of the equity market-capitalization-weighted portfolio optimally hedged against currency risk. A major result is that all investors should identically hedge their international investments, whatever their nationality or their level of risk aversion[4]. The optimal hedge ratio is therefore "universal". Unfortunately, the equilibrium hedge ratios depend on unobservable variables such as relative risk aversion and net foreign positions. Black (1990) simplifies the equilibrium model and comes up with a universal hedging rule. He estimates that international investments should be currency-hedged with a ratio of approximately 70%. Besides the difficulty of assessing the "exact" optimal hedge ratio, it appears that the normative implications of the global equilibrium model are not verified in the real world. Rather than holding the world market portfolio, investors exhibit a strong home bias in their equity portfolios.

In practice, many asset managers simply conduct an asset allocation mean-variance optimization based on expected returns and risk. Typically a two-step approach is implemented. The asset allocation to international asset markets is determined in a first step, and the amount of currency hedging is then decided for this specific asset allocation. So currency hedging is optimized assuming that the global asset allocation is fixed. If currency risk premia are nil[5] and if asset returns are uncorrelated with currency movements; then the optimal hedge ratio that minimizes risk is 100% because currency risk is pure noise (see Pérold and Schulman, 1988). So the currency-risk-minimizing strategy is a hedge ratio of 100%.[6] Again, this hedge ratio obtains for any (positive) level of risk aversion and should therefore be "universal". According to portfolio theory, deviations from this fully-hegded policy can only be explained by belief in the correlation between asset returns and currency movements and/or currency risk premia.

Behavioral finance claims that investors do not follow the traditional mean-variance paradigm and this is clearly applicable to the currency hedging decision (see Statman, 2005). We will rely on regret theory to propose an alternative theory of the currency hedging decision made by portfolio investors.

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[4] The risk level is adjusted by the combination of the risky portfolio and the national risk-free asset. While the optimal hedge ratio for a specific asset is identical for all investors, different assets could have different optimal hedge ratios.

[5] There is extensive theoretical and empirical discussion about the existence and magnitude of currency risk premia. This is beyond the scope of this paper. Given the difficulty in estimating a forward-looking long-term currency risk premium, most asset managers set a default value of zero.

[6] Of course, the optimal hedge ratio will differ from 100% if there is correlation between asset returns and currency movements, and if the currency risk premium differ from zero.

Regret is a cognitively-mediated emotion of pain and anger when, with hindsight, agents observe that they took a bad decision in the past and could have taken one with better outcome. In financial markets, agents will experience regret when their investment yields, ex-post, a lower performance than an obvious alternative decision that they could have chosen. Contrary to mere disappointment, which is experienced when a negative outcome happens relative to prior expectations, regret is an emotion strongly associated with a feeling of responsibility for the choice that has been made, based on a comparison of the actual outcome with the best outcome that could have been achieved.[7] Regret is such a powerful negative emotion that the prospect of its future experience may lead individuals to make seemingly sub-optimal, non-rational decisions relative to the expected utility paradigm. As the opening quote suggests, the anticipation of future regret was strong enough to turn Harry Markowitz away from his very own portfolio allocation theory when faced with a financial decision on his pension plan. We believe regret may also apply to sophisticated fund managers when currency risk is concerned.

Currency hedging is a dimension where regret clearly applies.[8] For example, an American investor who decided not to hedge currency risk would have incurred a currency loss of some 40% on its eurozone assets from late 1998 to late 2000, with a vast regret of not having fully hedged. Conversely a fully-hedged investor would have missed the 50% appreciation of the euro from late 2001 to late 2004. Again, a vast regret of not having taken the "right" hedging decision.[9]

There is an extensive literature in experimental psychology and, to a lesser extent, neurobiology that supports the assumption that regret influences decision-making under uncertainty[10] beyond disappointment and traditional uncertainty measures. Based on this concept of regret, Loomes and Sugden (1982) and Bell (1982) derived independently an economic theory of regret. They proposed a theory of choices under uncertainty that explains many observed violations of the axioms used to build the traditional expected utility approach. Regret theory (RT) assumes that agents are rational but base their decisions not only on expected payoffs but also on expected regret. It predicts Allais' paradox ("common consequences effect") and many other axiom violations reported in experiments by Kahneman and Tversky (1979) and others. RT bears some similarities with prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) as many results of RT are consistent with the empirical observations of human behavior that constitute the building blocks of prospect theory. But prospect theory is primarily descriptive while RT is a normative theory of rational choice under uncertainty (section 2 further discusses this issue). RT incorporates regret into the utility function in addition to the traditional value function of total wealth. Investors reach their investment decision by maximizing the expected value of this modified utility. So investors try to anticipate regret and take it into account in their investment decisions in a consistent manner. RT is clearly relevant to investors who compare the performance of their portfolio to foregone alternatives that they could have chosen, or to peers and benchmark portfolios whose performance could have been achieved.

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[7] See Mellers, Schwartz and Ritov (1999).

[8] See also Gardner and Wuilloud (1995), and Statman (2005).

[9] Furthermore, selling short an appreciating foreign currency leads to cash losses on the forward position that have to be covered by the sale of assets. A forced decision that is painful.

[10] For experimental psychology reviews see Gilovich and Medvec (1995), Zeelenberg et al., (2000). For neurobiological experiments see Damasio (1994), Bechara, Damasio and Damasio (2000), and Camille et al., (2004).

In this article, we use RT to account for the observed currency risk hedging behavior of fund managers with assets invested in foreign markets. While foreign currency hedging is an important decision in its own right, it also is simple enough to allow the modeling of regret in the utility function. We assume that the asset allocation has been chosen and focus on the currency hedging policy, as traditionally assumed in the hedging literature. Although, to our knowledge, this is the first attempt to apply[11] RT to currency hedging decisions, the experience of regret in currency hedging is not news for the investment world. Several practitioners have justified a 50% naïve hedge ratio on such intuitive grounds. For example:

> "A partial hedging policy – such as 50/50 or 70/30 – means the investor won't ever experience the major highs of an unhedged portfolio, but won't be subject to the lowest returns either."
> *"To Hedge or not to hedge", Simon Segal, SuperReview.com.au, 21 march 2003*
>
> "The 50% hedge benchmark is gaining in popularity around the world as it offers specific benefits. It avoids the potential for large underperformance that is associated with "polar" benchmark, i.e. being fully unhedged when the Canadian dollar is strong or being fully hedged when it is weak. This minimizes the "regret" that comes with holding the wrong benchmark in the wrong conditions."
> *" Managing Currency Risk: A Canadian Perspective", Gregory Chrispin, State Street Global Advisors, Essays and Presentations, March 23, 2004.*

The 50% hedge ratio is the simplest currency hedging policy that attempts to deal with regret. We propose a theory that encompasses all the cases of Figure 1 for different level of risk and regret aversion.

We will now propose a formal analysis of the optimal currency hedging decision that incorporates regret in an expected utility optimization, and therefore deals simultaneously with traditional risk (volatility of final wealth) and regret risk. The paper is structured as follows. In Section 2, we introduce RT and apply it to the modeling of currency hedging in Section 3. Section 4 derives closed-form hedging rules for currency risk minimization, while Section 5 derives closed-form hedging rules in the general case with expectations on currency movements and correlation between asset returns and currency movements. Section 6 concludes this paper.

2. Regret Theory

Regret theory (RT) developed by Loomes and Sugden (1982) and Bell (1982) is a theory of rational choice under uncertainty that is parsimonious yet can explain many of the observed axioms violations of traditional expected utility theory[12]. These authors derive a modified utility function of final wealth x resulting from a given investment choice, knowing that a different investment choice would have led to a final wealth y:

$$U(x,y) = v(x) + f(v(x) - v(y)) \tag{1}$$

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[11] The other applications of RT to financial decisions, that we are aware of, are Braun and Muermann (2004) and Dodonova and Khoroshilov (2005). The former apply regret theory to demand for insurance, the latter to asset pricing.

[12] As mentioned in the introduction, RT predicts Allais' paradox ("common consequences effect"), the "common ratio effect", the "isolation effect", the "preference reversal effect", the "reflection effect", and "simultaneous gambling and insurance".

where *U(x,y)* is the modified utility of achieving *x*, knowing that *y* could have been achieved. *v(x)* is the traditional utility function, also called value function or choiceless utility. It is the "value" or utility that an investor would derive from outcome *x* if he experienced it without having to choose. This value function is assumed to be monotonically increasing and concave (risk aversion) as in traditional finance. The difference *v(x) - v(y)* is the value loss/gain of having chosen *x* rather than a foregone choice *y*. It indicates the regret of having chosen *x*, when *y* could have been chosen. The regret function *f*(.) is monotonically increasing and decreasingly concave[13], with $f(0) = 0$. This modified utility *U*(.) is defined over the ex-post (final) outcomes of investment choices; and rational investors would make choices ex-ante by maximizing the expected value of this modified utility. Loomes and Sugden (1982, 1983) and Bell (1982, 1983) conclude that this is a well-behaved parsimonious functional form that allows to take regret into account and is consistent with empirically-observed deviations from traditional expected utility theory.

This functional form has been initially derived for pair-wise choices, but it can be extended (see Quiggin, 1994) to general choice sets. Consider that an investor can select among various investments *i* (e.g, some portfolio *i*), with outcome x_i. The modified utility of choosing investment *i* is given by:

$$U(x_i) = v(x_i) + f\left(v(x_i) - v(\max\left[x_i\right])\right) \qquad (2)$$

where $\max\left[x_i\right]$ is the best ex-post outcome that can be obtained among all investments. Note that the regret term $v(x_i) - v(\max\left[x_i\right])$ is always non-positive. Concavity of the regret function, $f''<0$, implies regret aversion.

Rational investors choose the optimal investment portfolio by maximizing their expected modified utility of all possible investment choices. So investors try to anticipate regret and take it into account in their investment decisions in a consistent manner.

It can be useful to highlight intuitively the difference with traditional expected utility. As opposed to traditional von Neumann- Morgenstern utility, which is only defined over the actual portfolio owned by the agent, the modified utility also includes a comparison with other portfolios that could have been chosen but are not currently owned by the agent. Regret-averse investors do take into account traditional value but also deviations from the (ex-post optimal) benchmark. Note that the benchmark is not a predetermined passive benchmark, but rather a benchmark that will be determined ex-post as the best-performing investment. So regret risk is different from tracking-error risk relative to a predetermined benchmark. In the mean-variance paradigm, investors care about the expected return and volatility of their portfolio. A regret-averse investor do care about the expected return and volatility of their portfolio, but in addition the investor also cares about the risk of deviations from the benchmark (regret risk). So there are two risk attributes in the utility function: volatility and regret risk.

With the observed evidence in favor of the influence of regret on decision-making under uncertainty[14] as well as the axiomatic and normative appeal of RT for investment choices, it is

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[13] Bell (1982, 1983) and Loomes and Sugden (1982,1983) show that several behavioral patterns which contradict traditional expected utility theory are predicted by regret theory with a function *f*(.) that is concave for negative values of the argument and with $f'''>0$, so that *f*(.) is decreasingly concave.

[14] Connolly and Zeelenberg (2002), page 212, state that "the emotion that has received the most research attention from decision theorists is regret".

surprising that RT has caught so little attention in the field of finance. Indeed, it is prospect theory that has been extensively used in behavioral finance. The success of prospect theory holds as much to its descriptive power as to the ability to handpick only some features that enable to explain selected puzzles in the field. Numerous authors have used prospect theory in normative models of investment choices, i.e. maximizing some expected utility (e.g. Benartzi and Thaler, 1995; Shefrin and Statman, 2000; Barberis, Huang and Santos, 2001, Berkelaar, Kouwenberg and Post, 2004, Gomes, 2005). But only a couple of features of prospect theory can be retained in such models. Utility models inspired by prospect theory typically include a disappointment term with a kink at the current investment value (the "reference point") where the slope of utility is higher for losses than for gains ("loss aversion"). Disappointment is easier to model[15] as the benchmark expectation for a given investment is usually set as a fixed number (possibly the current situation or the current position plus a risk-free rate)), while in RT the "best" investment strategy in the investment decisions universe can only be determined ex-post. In spite of the increased complexity, RT is clearly relevant to investors who compare the performance of their portfolio to foregone alternatives that they could have chosen, or to peers and benchmark portfolios whose performance could have been achieved.

It must be stressed that RT, although intuitively appealing, is difficult to apply because of the technical difficulties associated with the optimization of an expected utility function with two attributes: value and regret. Indeed, applying RT to a general portfolio problem involving numerous assets seems a daunting technical task. This is because, regret stems from a comparison of the actual return outcome of each portfolio with the actual return outcome of all other feasible portfolios.[16] In contrast, including disappointment in a utility function is much less intricate, as disappointment results from a comparison, for each asset independently, of the actual return outcome to a preset expectation return (e.g. zero, or the risk-free rate, or some other exogenous number). This technical difficulty probably explains the lack of applications of RT to the field of finance. However, currency hedging decisions are simple enough to model in the framework of RT as the ex-post optimal currency hedging choice is only one of two decisions: no hedging (if the foreign currency appreciated) or full hedging (if the foreign currency depreciated).

To summarize, a regret-theoretic approach presents an alternative way of introducing emotions in investment choices. While it suffers from the criticism that it only describes one aspect of human behavior, it does so in an elegant axiomatic way. Furthermore, it goes beyond modeling disappointment, as usually done in the literature, but deals with regret which seems an important psychological trait in portfolio choices, where investors care about the outcome of their choice relative to other strategies they could have followed, passive benchmarks and peers.

3. Currency Hedging: A Regret-Theoretic Framework

We consider that the currency hedging decision is a residual one, once the global asset allocation has been chosen. This is the approach traditionally taken in the hedging literature and, accordingly, we do not claim to solve simultaneously the general problem of individual

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[15] For example, Barberis and Huang (2004) focus on narrow framing and loss aversion by adding a piece-wise linear disappointment term for selected (narrow-framed) assets in a traditional utility function. There is a simple additive disappointment term for each of these assets. Loss aversion appears because the linear slope is higher for negative returns on these assets than for positive returns. The reference return is the risk-free rate.

[16] In mean-variance analysis, we only compare the expected returns and volatility of portfolios.

security selection, asset allocation and currency hedging. It is also consistent with the behavioral approach of mental accounting, the human tendency to treat each type of investment decisions in a separate mental compartment (also called "narrow framing")[17]. As described in Kahneman and Lovallo (1993), decision makers are excessively prone to treat problems as unique; their evaluation of single risky prospects neglects the possibility of pooling risks. Rather than looking at the whole portfolio as prescribed by traditional expected utility theory, investors tend to reach the best decision in each mental compartment. This feature is widely observed as far as the currency exposure decision is concerned. In global asset allocation, investors clearly separate the asset allocation decision and the currency risk hedging decision. Such a behavior is confirmed in a survey[18] of Canadian pension plans. The vast majority of these plans (94%) believe that the best way to handle currency exposure is to decide first on global asset allocation and then handle the currency exposure. This confirms that the currency hedging decision is indeed taken as a residual/separate decision from the investment decision that creates the currency exposure.

Of their initial wealth, W_0 , investors have allocated W_0^d to domestic asset and W_0^f to foreign assets:

$$W_0 = W_0^d + W_0^f$$

All valuations are conducted in domestic currency (e.g. the dollar for American investors).

As in all currency-hedging research, we do not focus on the interaction between domestic assets and foreign currency and will make the simplifying assumption that the final value of domestic assets, W^d, is non-stochastic. Introducing stochastic domestic return in our model, would simply add to the complexity of the notations[19], but the analysis would not be affected since the asset allocation decision is already fixed. The dollar value of foreign assets is equal to the product of the foreign-currency value of the foreign assets times the exchange rate (dollar value of foreign currency). Using log of price changes as return, the final (dollar) value of the foreign position, W^f, is:

$$W^f = W_0^f (1 + \tilde{R} + \tilde{s})$$

Where $\tilde{R}$ is the return of the foreign asset in its local currency and s is the percentage currency movement (e.g. changes in the dollar value of the foreign currency).

Investors decide to hedge a proportion h of the foreign assets against currency risk, by selling the foreign currency forward[20]. Foreign assets are treated as an homogeneous asset class with a single currency. This is equivalent to saying that American investors care about an appreciation of the dollar against all currencies (a drop in the weighted average dollar value of foreign currencies, where the weights are those of the selected foreign asset allocation). A hedge ratio of zero implies no currency hedge and a hedge ratio of one implies full currency

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[17] For a discussion of narrow framing, or mental accounting, see Tversky and Kahneman (1981), Shiller (1999), Thaler (1999), Shefrin and Statman (2000), Kahneman (2003), Barberis and Huang (2004).

[18] William M. Mercer Investment Consulting's *Survey of Pension Plans On Currency Issues*, September 2000 (conducted in 2000 with responses from more than 100 large funds).

[19] $\tilde{R}$ in equation (4) would simply be replaced by the weighted-average return on the global allocation.

[20] We assume that interest rates are equal worldwide, so that the forward exchange rate is equal to the spot exchange rate.

hedging. As forward contracts have a zero initial value, the initial wealth is unchanged by the hedging decision. Given a hedge ratio *h*, the final wealth value is given by:

$$W = W^d + W_0^f(1+\tilde{R}+\tilde{s}) - hW_0^f\tilde{s}$$

$$W = W^d + W_0^f(1+\tilde{R}+\tilde{s}[1-h]) = W^H + W_0^f\tilde{s}(1-h) \qquad (3)$$

where W^H refers to final wealth with full hedging.

For our purpose, the global asset allocation is fixed. Hence, the value (traditional utility) of final wealth, *V(W)*, can be written as a function of *h*, the sole decision variable, and of the two stochastic variables $\tilde{R}$ and $\tilde{s}$:

$$V(W) = v(\tilde{R}+\tilde{s}[1-h])$$

Note that derivatives satisfy the conditions $v' = W_0^f \times V'$ and $v'' = W_0^f \times W_0^f \times V''$.

The modified utility can be written as:

$$U(W) = u(h,\tilde{R},\tilde{s}) = v(\tilde{R}+[1-h]\tilde{s}) + f(v(\tilde{R}+[1-h]\tilde{s}) - v(\tilde{R}+\max[[1-h]\tilde{s}])) \qquad (4)$$

Where *v*(.) and *f*(.) are monotonically increasing and concave; *f*(.) is decreasingly concave ($f''<0$, $f'''>0$) and $f(0)=0$.

We assume that investors only exhibit regret on the currency dimension, not on the asset allocation. This is a natural assumption of our model, as investors only optimize the currency hedging dimension. In addition experiments (Harless, 1992; Camille et al., 2004) show that regret is only experienced if the outcome of unchosen options are "visible" (or "accessible"), and currency returns are highly visible and emotional. Indeed, media talk daily about the fate of the dollar. In addition, all performance reports separate the currency gains/losses on the portfolio from other sources of return. Performance relative to peers or other simple hedging strategies are important. Furthermore, everyone, even outside the sphere of finance, seems to have an opinion on the value of the dollar, especially ex-post. What might have been a reasonable hedging decision ex-ante, can be easily criticized ex-post by a board of trustees.

Currency hedging is easy to analyze within a regret-theoretic approach because, with hindsight, the optimal hedging decision can only be one of two possible choices. If the foreign currency appreciated, and whatever the positive value of *s*, the best hedging policy would have been to stay unhedged (*h*=0). So for any positive *s*:

$$\max[[1-h]s] = s$$

If the foreign currency depreciates by any amount, the best hedging policy would have been to be fully hedged (*h*=1). So for any negative *s*:

$$\max[[1-h]s] = 0$$

Equation (4) can be written as:

$$u(h,\tilde{R},\tilde{s}) = v(\tilde{R}+[1-h]\tilde{s}) + f_{s+}(v(\tilde{R}+[1-h]\tilde{s}) - v(\tilde{R}+\tilde{s})) + f_{s-}(v(\tilde{R}+[1-h]\tilde{s}) - v(\tilde{R})) \qquad (5)$$

Let us focus on the impact of a currency movement *s*. The utility *u*(.) is continuous and twice differentiable except in *s*=0. At *s*=0, the left-hand derivative with respect to *s* is equal to:

$$\frac{\partial u}{\partial s} = (1-h)v'(\tilde{R}) + (1-h)f'(0)v'(\tilde{R})$$

The right-hand derivative is equal to:

$$\frac{\partial u}{\partial s} = (1-h)v'(\tilde{R}) - hf'(0)v'(\tilde{R})$$

At s=0, the slope on the negative side is greater than on the positive side, as the difference $f'(0)v'(\tilde{R})$ is always positive. As a result, the utility function u(.) presents a kink at $s = 0$. Furthermore, the function u(.) is concave with respect to s (see Appendix A). The current exchange rate is a reference point and investors are more sensitive to reductions in financial wealth than to increases in financial wealth. These are common features in prospect theory. Here we have "currency loss aversion", to coin a term frequently used in behavioral finance. This results from the regret aversion on currency losses.

In Figure 2, we illustrate the modified utility with regret aversion by assuming two simple functional forms for v(.) and f(.). Investors have a logarithmic value function and $f(x)$ is a negative exponential in the form $1-e^{-X}$. For this illustrative purpose focusing on currency risk, we assume that the return on foreign assets R is non-stochastic. Without loss of generality, we take R=0. Wealth is assumed constant except for the impact of a currency movement from the current exchange rate. So the utility[21] depicted in Figure 1 is a function of the exchange rate movements but also of the hedging decision that is taken. To contrast this modified utility with the traditional value function, we also plot the utility function without regret in dashed line.

Let us first consider Figure 2a which assumes that no currency hedging takes place (h=0). For positive values of s, utility increases with s because of the increase in the value function and the absence of regret (no hedging is ex-post optimal when the foreign currency appreciates). For negative values of s, utility decreases with -s because of the decrease in the value function but also because investors experience regret of not having hedged. Let us now consider the case, illustrated in Figure 2b, where the investor undertakes some currency hedging (h=0.5). For positive values of s, on the one hand, the value function increases with s because of the wealth increase caused by 50% of the currency appreciation (half of it is hedged), but there is regret to have only half of the position hedged while the currency appreciates. As a result, the slope becomes smaller than for the utility function without the regret term. For negative values of s, the investor suffers a reduction in wealth (smaller than when h=0), but also a regret to have only hedged 50% of the position when it would have been optimal to have hedged fully. Figure 2c illustrates the case of full hedging. For positive values of s, the investor's wealth remains unchanged (full hedge), but he experiences regret to have missed the currency appreciation. For negative values, the value function remains unchanged (as the currency depreciation is fully hedged) and there is no regret as the optimal hedging decision was taken. Note that in all three cases there is currency loss aversion, with a kink at s=0 : the slope of the utility function is larger for negative values of s than for positive.

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[21] The combination of the two functions yields a very simple form for the modified utility. It is equal to a constant plus: $\log(1+(1-h)s)-1/(1+(1-h)s)$ for negative values of s and $\log(1+(1-h)s)-1/(1+(1-h)s)-s/(1+(1-h)s)$ for positive values of s.

Figure 2: Utility function with regret aversion for various hedging decisions

Utility is given as a function of s. Dashed lines represent value function alone while full lines represent utility function with regret aversion

Figure 2a: *h*=0 (No hedging)

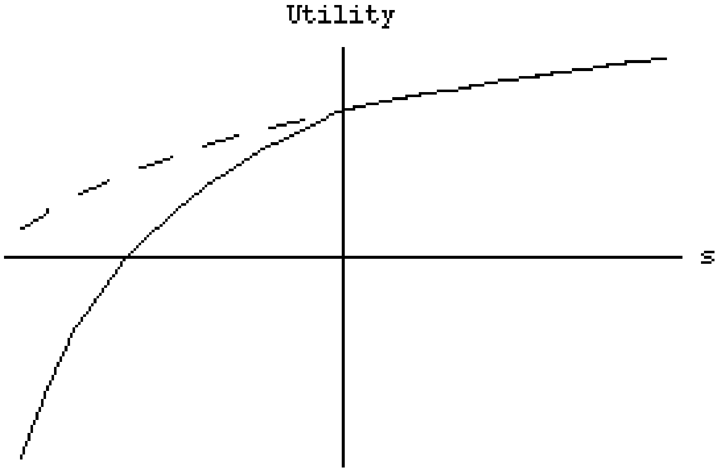


Figure 2b: *h*=0.5 (Partial hedging)

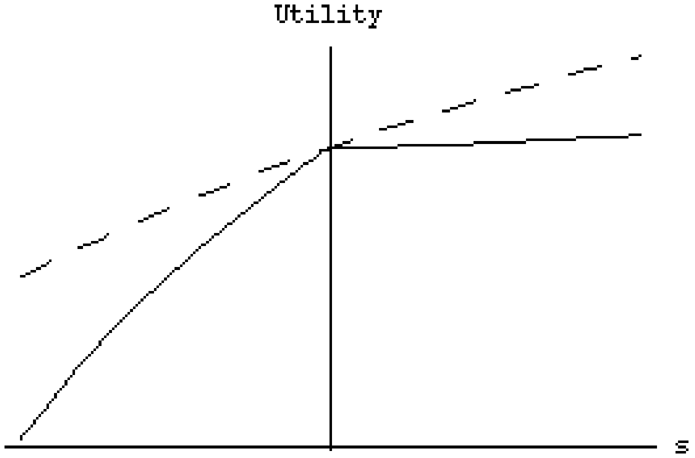
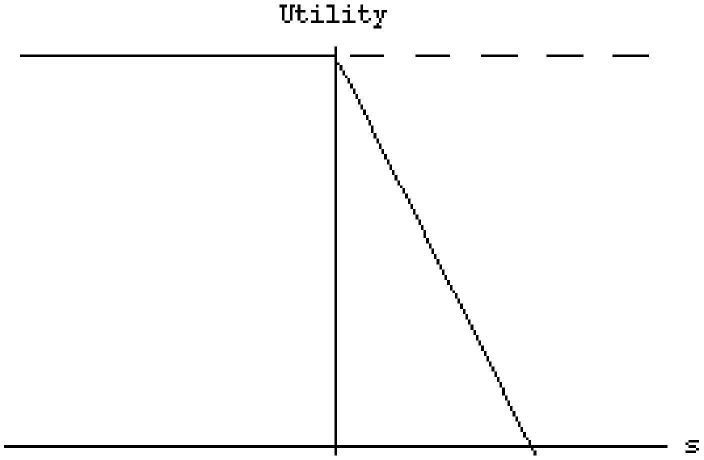


Figure 2c: *h*=1 (Full hedging)



4. Derivations and Results: Currency risk minimization

The optimal hedge ratio is obtained by maximizing the expected modified utility with respect to h. It can be noted that $u(h,R,s)$ in (5) is concave with respect to h (see Appendix B). To derive optimal hedging rules, we need to make specific assumptions on the functions $v(.)$ and $f(.)$ to be used as well as on the distribution of s. If $f(.)$ is linear (no regret aversion), then the problem reduces to traditional expected utility maximization, as the maximization with respect to h of the expected utility given in (5) reduces to the maximization of $Ev(\tilde{R}+\left[1-h\right]\tilde{s})$. With a linear regret function, RT always reduces to traditional expected utility theory.

In general $f(.)$ is assumed concave (regret aversion). Except for very particular and simplistic functions[22] $v(.)$ and $f(.)$, we cannot derive explicit hedging rules and would have to resort to numerical solutions with little generality. The problem already arises in the case of maximizing expected traditional utility in portfolio theory, but there exist some interesting cases where explicit rules can be worked out.[23] In our model, the problem is compounded by the presence of a piece-wise regret function defined over a value function. An ad-hoc assumption, that would make the model a bit more tractable, could be to model the regret term as piece-wise linear and defined over payoffs, not valuation of payoffs.[24] But this simplification would not be consistent with RT and we would lose the theoretical and empirical appeal of this approach.

An interesting alternative is to use the two-moment approximation proposed by Pratt (1964) to conduct his analysis of risk aversion for small risks. We use a Taylor expansion of (5) and take its expected value, ignoring moments higher than two. We then maximize with respect to h and are able to derive explicit hedging rules with interesting economic interpretation. This two-moment Arrow-Pratt approximation is very similar in spirit and results to the multivariate normality assumption for return distribution that was introduced in the finance literature[25]. In both cases, we end up with models relying solely on the first two moments of return distributions. In traditional finance models, the normality assumption implies that for well-behaved utility functions, expected utility $Eu(.)$ can be expressed as a function of the means and covariances. So parameters of the utility function only affect investment choices to the extent that they affect the risk aversion parameter in the expected utility function. In our model, the modified utility function is complex with two attributes, risk and regret. To allow for economic interpretation, we wish to explicitly retain the parameters of the modified utility in the optimal hedging rules derived under maximization of the expected modified utility. This cannot be done by simply assuming normality of returns. However, we can do it in the

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[22] An unattractive alternative is to assume that v is linear (no risk aversion) and f quadratic.

[23] When the utility function belongs to the HARA class and asset returns are multivariate normally distributed, there is a linear relation between optimal portfolio weights and the wealth level.

[24] So $f(v(R+\left[1-h\right]s)-v(R+\max\left[\left[1-h\right]s\right])$ would be replaced by $-hs$ for positive values of s and $(1-h)s$ for negative values of s.This would be similar in spirit to the piecewise linear simplification in the disappointment approach of Barberis and Huang (2001, 2004). It would imply that the regret term is only concave at the kink s=0.; investors would not exhibit regret aversion elsewhere. For example a loss of $2x$ is just twice as unpleasant as a loss x. One can check that the risk-minimizing hedge ratio would not be affected by regret and would be similar to what obtains under traditional expected utility optimization (a hedge ratio of one).

[25] Or lognormality in the case of continuous-time models,.

case of the Arrow-Pratt approach.[26] Most of the hedging literature has been using the two-moment assumption of multivariate normal distributions for $\tilde{R}$ and $\tilde{s}$, where the first two moments of the distributions are sufficient to characterize the whole distributions. As we will compare our results to this traditional mean-variance optimization, we are quite satisfied with making an equivalent two-moment assumption.

As mentioned before, our primary focus is on the currency-risk-minimizing behavior of investors, where investors search for the optimal hedge ratio in the absence of expectations on currency movements and of specific assumptions about the correlation between foreign asset returns and currency movements. Hence, we will now detail the derivations under the simplifying assumptions that the return on foreign assets is non-stochastic that the expected currency return is zero, and that the distribution is symmetric. To simplify notations, and without loss of generality, we set R=0. We will provide a discussion relaxing those assumptions in the next section.

The expectation of (5) under those assumptions can be written as:

$$Eu = Ev([1-h]\tilde{s}) + E_{s+} f(v([1-h]\tilde{s}) - v(\tilde{s})) + E_{s-} f(v([1-h]\tilde{s}) - v(0)) \qquad (6)$$

As mentioned above, our problem is well-behaved as first derivatives of u(.) are well-defined and continuous, except in s=0, and u(.) is concave in h and s. Eu(.) is concave in h as shown in Appendix B.

The optimal hedge ratio satisfies the first order condition:

$$\frac{\partial Eu}{\partial h} = 0$$

Because Eu(.) is concave in h, this first-order condition is necessary and sufficient for optimality.

In the *absence of regret aversion* (f(.) is linear) the optimization problem reduces to the traditional expected utility optimization $Max_h Ev([1-h]\tilde{s})$. Here $\tilde{s}$ is a pure risk (no expected return) and 1-h is non negative, therefore any risk averse investor will attempt to eliminate that risk by setting h equal to 1. This is the typical full-hedging risk-minimization result.

We now derive the Arrow-Pratt approximation in the *presence of regret aversion* (f''<0).

For a given hedge ratio h, the Taylor expansion[27] of $v([1-h]\tilde{s})$ around s=0 is:

$$v\left([1-h]\tilde{s}\right) = v(0) + (1-h)\tilde{s}v'(0) + \frac{1}{2}(1-h)^2\tilde{s}^2 v''(0) + o(\tilde{s}^2)$$

Hence, the expected value function:

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[26] Strictly speaking, the Arrow-Pratt approximation is valid for small risks. The quality of the two-moment approximation depend on the actual return distributions and the shape of the utility function. This has been extensively discussed in the literature, see Samuelson (1970), Loistl (1976), Levy and Markowitz (1979), and Kroll, Levy and Markowitz (1984). We thank Christian Gollier for his support in getting a clearer view of this approach.

[27] Our derivations could be made a bit more formal by taking $s=\xi s'$ and letting ξ become very small, where s' is a normal distribution. This is a direct application of the "compact" derivations of the approximation by Samuelson (1970).

$$Ev\left(\left[1-h\right]\tilde{s}\right)\approx v(0)+(1-h)E(\tilde{s})v'(0)+\frac{1}{2}(1-h)^2E(\tilde{s}^2)v''(0) \tag{7}$$

The expected regret function, over $s+$:

$$E_{s+}f(v(\left[1-h\right]\tilde{s})-v(\tilde{s}))\approx f(0)+E_{s+}\left[v(\left[1-h\right]\tilde{s})-v(\tilde{s})\right]f'(0)+\frac{1}{2}E_{s+}\left[v(\left[1-h\right]\tilde{s})-v(\tilde{s})\right]^2 f''(0)$$

over s-:

$$E_{s-}f(v(\left[1-h\right]\tilde{s})-v(0))\approx f(0)+E_{s-}\left[v(\left[1-h\right]\tilde{s})-v(0)\right]f'(0)+\frac{1}{2}E_{s-}\left[v(\left[1-h\right]\tilde{s})-v(0)\right]^2 f''(0)$$

With:

$$E(v\left(\left[1-h\right]\tilde{s}\right)-v(\tilde{s}))\approx -hE(\tilde{s})v'(0)+\frac{1}{2}\left[(1-h)^2-1\right]E(\tilde{s}^2)v''(0)$$

$$E(v\left(\left[1-h\right]\tilde{s}\right)-v(0))\approx (1-h)E(\tilde{s})v'(0)+\frac{1}{2}(1-h)^2E(\tilde{s}^2)v''(0)$$

Let's drop the argument 0 in the derivatives. Let's denote $\overline{s}=E(\tilde{s})=0,\ \overline{s}_+=E_{s+}(\tilde{s}),\ \overline{s}_-=E_{s-}(\tilde{s}),\ \Sigma_s=E(s^2),\ \Sigma_{s+}=E_{s+}(\tilde{s}^2),\ \Sigma_{s-}=E_{s-}(\tilde{s}^2)$. Note that $f(0)=0$, $\overline{s}=\overline{s}_++\overline{s}_-=0$ and that $\Sigma_s=\sigma_s^2=\Sigma_{s+}+\Sigma_{s-}$. With symmetric (e.g. normal) distributions we have $\Sigma_{s+}=\Sigma_{s-}=\frac{1}{2}\Sigma_s$.

The expected value function (7) becomes:

$$Ev\left(\left[1-h\right]\tilde{s}\right)\approx v(0)+\frac{1}{2}(1-h)^2\Sigma_s v'' \tag{8}$$

Discarding moments higher than two, we get for the expected regret function over $s+$:

$$E_{s+}f(v(\left[1-h\right]\tilde{s})-v(\tilde{s}))\approx\left[-h\overline{s}_+v'+\frac{1}{2}\left[(1-h)^2-1\right]\Sigma^+v''\right]f'+\frac{1}{2}h^2\Sigma_{s+}v'^2\ f'' \tag{9}$$

Similarly for s-:

$$E_{s-}f(v(\left[1-h\right]\tilde{s})-v(0))\approx\left[(1-h)\overline{s}_-v'+\frac{1}{2}(1-h)^2\Sigma_{s-}v''\right]f'+\frac{1}{2}(1-h)^2\Sigma_{s-}v'^2\ f'' \tag{10}$$

The expected utility is the sum of three terms:

$$Eu = (8) + (9) + (10) \tag{11}$$

The expected utility (11) can be rewritten by grouping the terms in various powers of h as:

$$\begin{aligned}Eu &\approx v(0)+\frac{1}{2}v''\Sigma_s+v'f'\overline{s}_-+\frac{1}{2}f'v''f'\Sigma_{s-}+\frac{1}{2}v'^2\ f''\Sigma_{s-}\\ &-hv''\Sigma_s-hv''f'(\Sigma_{s+}+\Sigma_{s-})-hv'^2\ f''\Sigma_{s-}\\ &+\frac{1}{2}h^2v''\Sigma_s+\frac{1}{2}h^2v''f'(\Sigma_{s+}+\Sigma_{s-})+\frac{1}{2}h^2v'^2\ f''(\Sigma_{s+}+\Sigma_{s-})\end{aligned} \tag{12}$$

Where $\Sigma_{s+} + \Sigma_{s-} = \Sigma_s$.

We now compute the first order condition for optimal hedging by setting the derivative of *Eu*(.) with respect to *h* equal to zero.

$$0 = -v''\Sigma_s - v'' f'\Sigma_s - v'^2 f''\Sigma_{s-} + hv''\Sigma_s + hv'' f'\Sigma_s + hv'^2 f''\Sigma_s \qquad (13)$$

Hence:

$$h^* = \frac{v''(1+f')\Sigma_s + v'^2 f''\Sigma_{s-}}{v''(1+f')\Sigma_s + v'^2 f''\Sigma_s} = \frac{v''(1+f') + v'^2 f''(\Sigma_{s-}/\Sigma_s)}{v''(1+f') + v'^2 f''}$$

We can rearrange h^*, noting that $\Sigma_{s-} = \frac{1}{2}\Sigma_s$:

$$h^* = 1 - \frac{1}{2}\frac{v'^2 f''}{v''(1+f') + v'^2 f''} = 1 - \frac{1}{2}\theta \qquad (14)$$

The optimal hedge ratio is equal to one, as would obtain in risk minimization without regret, minus a term linked to regret aversion. Note that *v*' and *f*' are positive (investors prefer more wealth and less regret), *v*" is negative (risk aversion) and *f*" is also negative (regret aversion), so θ is generally positive and lesser than one.

Let's now introduce traditional risk aversion $\lambda = -v''/v'$ and, following Bell (1983), define regret aversion ρ as:

$$\rho = \frac{-v' f''}{1+f'} \qquad (15)$$

We can now rewrite θ as :

$$\theta = \frac{\rho}{\rho + \lambda}.$$

As mentioned previously, when the regret function *f*(.) is linear (*f*"=0), we are back to traditional utility maximization and an optimal hedge ratio of 1. Since risk aversion and regret aversion are both positive, the optimal hedge ratio is always between 50% and 100%. Ceteris paribus, the lower the regret aversion ρ, the higher the optimal hedge ratio. When regret aversion ρ is very small relative to risk aversion λ (value dominates regret and θ is close to zero), h^* goes to 1 as ρ goes to 0, as can be expected from traditional expected utility theory. Conversely, when regret aversion is large relative to risk aversion (θ close to one), the optimal hedge ratio gets close to 50%. We call infinite regret aversion the case where regret aversion is very large relative to risk aversion.[28]

With this assumption of infinite regret aversion, we believe that our regret-theoretic model yields similar results to the “minimax regret” decision rule of Savage (1954). In this early model, Savage assumed that agents consider, for all possible decisions, the maximum regret that they may carry ex-post. Agents then select the decision that carries the smaller such “maximum regret”. Note that this decision is taken irrespective of the likelihood that such a

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[28] Note that this does not necessarily imply that ρ is infinite. It could also be that regret aversion dominates traditional risk aversion and that ρ is finite but λ is equal to zero. What really matters is the ratio ρ/λ.

regret may actually occur (provided that the probability is strictly positive). Intuitively, in our model with infinite regret aversion, investors care exclusively about the higher level of regret attained for any hedging decisions, whether in the region of gains or in the region of losses. When they hedge fully, investors anticipate that the maximum regret associated with a strong appreciation of the foreign currency, though unlikely, is so high that they reject such a hedging decision. Conversely, if they do not hedge at all, the regret associated with a strong depreciation in the foreign currency is again perceived as extremely high, even though it may be very unlikely, and is again rejected. As we assumed that the distribution of the foreign currency value is symmetric, the naïve 50% hedging policy will always be wrong and exhibit regret ex-post. However, the maximum amount of regret will be cut in half whether it is attained in the region of gains or in the region of losses.

To summarize the case of pure currency-risk minimization, a regret averse investor will always hedge less than 100%, the optimum hedging of a traditional expected-utility maximizer. However an optimal hedge ratio of 50% will only obtain for infinite regret aversion. In general the optimal hedge ratio will be between 100% and 50%, depending on regret aversion.

5. Derivations and Results: General case

We will now consider the general case where the return on foreign assets is stochastic and where the expected currency return can be non-zero. The derivations follow the previous methodology and are given in the Appendix C. The expected value to be maximized with respect to *h* is:

$$\begin{aligned} Eu(h,\tilde{R},\tilde{s}) &= Ev(\tilde{R}+\left[1-h\right]\tilde{s}) + E_{s+} f(v(\tilde{R}+\left[1-h\right]\tilde{s}) - v(\tilde{R}+\tilde{s})) \\ &+E_{s-} f(v(\tilde{R}+\left[1-h\right]\tilde{s}) - v(\tilde{R})) \end{aligned} \qquad (16)$$

where $\tilde{R}$ and $\tilde{s}$ are stochastic with mean $\bar{R}$ and $\bar{s}$, so that $\tilde{R} = \bar{R} + \tilde{r}$, where $\tilde{r}$ is a random variable with zero mean, $\Sigma_s = E(\tilde{s}^2)$ and $\Sigma_r = E(\tilde{r}^2)$.

Traditional utility with no regret

To compare with the existing literature on hedging, let's first consider the special case where there is no regret. Then for small risks[29], we get by developing around $\bar{R}$:

$$Eu(h,\tilde{R},\tilde{s}) \approx E(\tilde{r}+\left[1-h\right]\tilde{s})v'(\bar{R}) + \frac{1}{2}E(\tilde{r}+\left[1-h\right]\tilde{s})^2 v''(\bar{R})$$

$$Eu(h,\tilde{R},\tilde{s}) \approx \left[1-h\right]\bar{s}v'(\bar{R}) + \frac{1}{2}\left[\Sigma_r + 2(1-h)\operatorname{cov}(\tilde{r},\tilde{s}) + (1-h)^2\Sigma_s\right]v''(\bar{R}) \qquad (17)$$

The optimal hedge ratio is obtained by setting to zero the derivative of (17) with respect to *h*. We obtain:

$$0 = -\bar{s}v'(\bar{R}) + \left[-\operatorname{cov}(\tilde{r},\tilde{s}) - \Sigma_s + h\Sigma_s\right]v''(\bar{R})$$

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[29] The derivations can be made more formal by setting $s=\xi s'$, $R=\xi R'$ and letting ξ become very small, where s' and R'' are multivariate normal distributions. This is the spirit of the approach of Samuelson (1970).

$$h^* = 1 + \frac{\bar{s}}{\Sigma_s} \cdot \frac{v'(\bar{R})}{v''(\bar{R})} + \frac{\text{cov}(\tilde{r}, \tilde{s})}{\Sigma_s} = 1 - \frac{\bar{s}}{\Sigma_s} \cdot \frac{1}{\lambda} + \frac{\text{cov}(\tilde{r}, \tilde{s})}{\Sigma_s} \tag{18}$$

Where λ is the Arrow-Pratt measure of local risk aversion, $-v''/v'$. This result is the traditional one in the hedging literature. We will refer to it as the mean-variance case. It would be exact if the value function was quadratic or the distribution multivariate normal. Ceteris paribus, a positive expectation on the foreign currency movement reduces the optimal hedge ratio (speculative term). The lesser the risk aversion, the lower the hedge ratio (investors speculate more). Similarly, a negative covariance[30] between foreign asset return and currency movement (the local price of the foreign asset tends to go up when the foreign currency depreciates) reduces the optimal hedge ratio (covariance term).

Modified utility with regret

The derivations for the general case in presence of regret is given in Appendix C. The optimal hedge ratio in the general case is:

$$h^* = 1 - \frac{\Sigma_{s+}}{\Sigma_s} \times \frac{\rho}{\rho + \lambda} - \frac{\bar{s}}{\Sigma_s} \times \frac{1}{\rho + \lambda} + \frac{\text{cov}(r, s)}{\Sigma_s} \times \frac{\lambda}{\lambda + \rho} \tag{19}$$

The hedge ratio is equal to 100% (the risk-minimizing term) minus three terms:

a) *regret term*:

$$h_{regret} = -\frac{\Sigma_{s+}}{\Sigma_s} \times \frac{\rho}{\rho + \lambda} \tag{20}$$

This is similar to the expression $-\frac{1}{2}\theta$ of (14) in the currency risk-minimizing case, except that Σ_{s+} will generally differ from $\frac{1}{2}\Sigma$, if the expectation of *s* differs from zero[31]. Hence, the previous discussion applies with one caveat. If $\bar{s} > 0$, Σ_{s+} will be greater than $\frac{1}{2}\Sigma$ (for a symmetric distribution) and investors will hedge less because they anticipate to experience less regret if they decide not to hedge than in the risk-minimizing case. This is because they use the current exchange rate as reference point, while they anticipate the future exchange rate to appreciate. Conversely they would hedge more if they anticipate the foreign currency to depreciate.

b) *speculative term*:

$$h_{specul} = \frac{\bar{s}}{\Sigma_s} \times \frac{v'(1+f')}{v''(1+f') + v'^2 f''} = -\frac{\bar{s}}{\Sigma_s} \times \frac{1}{\rho + \lambda} \tag{21}$$

As in the traditional mean-variance case (18), a positive expectation on the foreign currency movement reduces the optimal hedge ratio. The lesser the risk aversion λ, the lower the hedge

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[30] The term $\frac{\text{cov}(\tilde{r}, \tilde{s})}{\Sigma_s}$ can be thought as the elasticity, or beta, of asset returns to currency movements.

[31] This would also be the case if the distribution of *s* is not symmetric.

ratio (investors speculate more). But this is a modified risk aversion that takes regret into account. Regret aversion will, overall, add to traditional risk aversion:

Without concavity in the regret function *f*, the risk aversion would be similar to the traditional one. In general, regret adds to risk aversion because *f''* is negative. Ceteris paribus, regret-averse investors will tend to "speculate" less on their anticipations of currency movements. However, as argued above, this effect will be mitigated by the regret term.

c) covariance term

$$h_{\mathrm{cov}} = \frac{\mathrm{cov}(\tilde{r},\tilde{s})}{\Sigma_s} \times \frac{v''(1+f')}{v''(1+f') + v'^2 f''} = \frac{\mathrm{cov}(\tilde{r},\tilde{s})}{\Sigma_s} \times \frac{\lambda}{\lambda+\rho} \qquad (22)$$

Where $\gamma = \lambda/(\lambda+\rho)$ is positive. In the trivial case of a linear regret function, $\gamma = \lambda/(\lambda+\rho) = 1$ and the covariance hedging term is identical to that in the traditional mean-variance case as shown in (18). In general, investors will take into account the covariance between asset return and currency movement, or more precisely the elasticity of asset return to currency movement, in their hedging decision. However, in the presence of regret, γ is less than one and investors tend to deviate less from their risk-minimizing hedging policy. The intuitive explanation is straightforward. A negative correlation between foreign asset return and currency movement implies that asset returns tend to soften the impact of currency risk at the portfolio level; but regret is only measured on the currency movement itself, not on asset return. The value function in the modified utility takes into account total portfolio risk and suggests a lower hedge ratio because of the negative correlation, but this is partly dampened by regret aversion on currency losses.

Note that with infinite regret aversion, the speculative and covariance terms are equal to zero, and the optimal hedging policy is a 50% hedge ratio. In the limit, with infinite regret aversion, the covariance term will no longer influence the hedging decision. This suggests that our result from previous section on infinite regret averse investors is robust to assumptions on the covariance between foreign assets and currency movement and on the expected movements of the foreign currency.

Explaining various hedging policies

Let us now return to the question asked in the introduction. How can our model explain the various hedging policies observed in Figure 1:

- A proportion of investors adopt a full hedging policy. These are traditional investors experiencing no regret. Currency expectations do not affect their benchmark policy and they consider that the covariance between asset returns and currency movement is null.
- A significant proportion of investors adopt a 50% hedging policy. They could be regarded as investors with infinite regret aversion. As far as currencies are concerned, they wish to minimize regret. As discussed above, these investors try to minimize the size of ex-post regret.
- The no-hedging policy can have several explanations. Our model suggest two possible explanations that can also be advanced for traditional investors.[32] First investors could

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[32] Because currency returns are a significant component of the total return on foreign assets, it is not surprising to remain unhedged on foreign assets that have been selected to enhance return. But it is clear that explanations for a no-hedging hedge ratio can also be found outside the scope of our model. Some investors are not allowed to

hold expectations that their own currency is overvalued and that foreign currencies will appreciate in the long run. If the expectation is large enough, this will suggest keeping a full currency exposure on foreign assets because of the speculative term (21) and the regret term (20). Another traditional explanation is that investors consider equities as "real" assets. Consider for example a high-inflation country where all prices, including nominal equity prices, go up at the rate of inflation. Then, the local currency also depreciates at that inflation rate. This would suggest a strong negative correlation between r and s and the term $\frac{\text{cov}(\tilde{r},\tilde{s})}{\Sigma_s}$ in the covariance term (22) would be equal to minus one. Investors with no regret ($\rho = 0$) would chose a zero hedge ratio in the absence of currency expectations as can be seen in equation (18). Indeed, Froot (1993) suggests that inflation gets built up in exchange rates and equity prices over the long run, implying a strong negative covariance term.

- Investor who adopt another hedging policy (13% of all investors) could do so because they have some level of regret aversion between zero and infinity.

The interaction between risk and regret aversion

We now discuss how the relative levels of regret and risk aversion affect hedging policies.

We provide in Figure 3 the optimal hedge ratio and its three components (regret term, speculative term and covariance term) for different levels of regret aversion. We arbitrarily set risk aversion at one, and let regret aversion vary from zero to 15. As an illustration we use different assumptions on market expectations. In all cases, the standard deviation of percentage currency movements is set at 10% per year (so $\Sigma_s = 1\%$). Figure 3a, presents the case where investors hold no expectations about currency movements so that $\bar{s} = 0$ and $\text{cov}(\tilde{r},\tilde{s}) = 0$. As discussed previously, we find that the risk-minimizing hedge ratio is 100% in the absence of regret aversion. The optimal hedge ratio reaches 75% when regret aversion equals risk aversion and drops to 50% when regret aversion dominates risk aversion.

Figure 3b introduces a positive expectation on the foreign currency with $\bar{s} = 1\%$ per year. In the absence of regret, the optimal hedge ratio[33] is 0%. As regret aversion increases, the optimal hedge ratio increases. This is caused by a strong increase in the speculative term in (21) as ρ increases: remaining unhedged creates the potential for a strong regret that is not offset by the utility of the higher expected return. The optimal hedge ratio is 25% when regret aversion equals risk aversion, and reaches 50% when regret aversion dominates risk aversion.

Figure 3c introduces a positive expectation on the foreign currency as well as a negative covariance between asset return and currency. In other words, the price of the foreign asset, measured in foreign currency, tends to rise when the foreign currency depreciates. This is an attractive feature from a risk viewpoint as it reduces the impact of a currency loss; it leads to a

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use derivatives or find the administrative costs and risks of hedging not worth it. Others wish to get a currency exposure because currencies bring an element of diversification to their domestic portfolio. This is typically the case if foreign assets represent a small portion of the overall portfolio, see Jorion (1999). Finally, the data in Figure 1 are benchmarks assigned to currency overlay managers not the actual "final" hedge ratio. When the benchmark is no hedging, a currency overlay manager might engage in currency-hedging to attain both a lower risk level and a higher return than the benchmark.

[33] The ratio of expected currency movement to variance is equal to one when $\bar{s} = \Sigma_s = 1\%$. For a risk aversion of one, this translates into a speculative term of –100% for the hedge ratio.

covariance term that reduces the amount of optimal hedging. In the simulation, the expected currency appreciation is 1% and the elasticity of asset return to currency movement is –0.2. In the absence of regret, investors would adopt a no hedging policy; actually they would even go long (speculate) on the foreign currency (optimal hedge ratio of –20%). A regret averse investor with a regret aversion similar to risk aversion would only hedge partially (15%); but the optimal hedge ratio would reach 50% when regret aversion dominates risk aversion.

While the simulations have been conducted for agents with different levels of regret aversion but a fixed level of risk aversion, it could be misleading to consider regret and risk aversion independently. The two attributes are likely to have some degree of substitutability. For example, an agent with high regret aversion could have lower risk aversion and vice versa.

Looking at these results, a natural question arises as to whether regret aversion can be observationally equivalent to risk aversion. In other words, can a risk averse investor ($\rho = 0$ and a risk aversion λ_0) exhibit the same hedging behavior as a regret averse investor (with some ρ and λ)? Because ρ and λ enter the three hedging terms in a different fashion ($\frac{\rho}{\rho+\lambda}$, $\frac{\lambda}{\lambda+\rho}$ and $\frac{1}{\rho+\lambda}$), a traditional investor with no regret aversion would not adopt the same hedging rule as a regret averse investor. In other words, we cannot fin a local risk aversion λ_0 that would yield the optimal hedging rule described above: for each set of market expectations (currency expected return and variance) we would get a different value of λ_0. But local risk aversion is a property of the utility function defined over current wealth, it is not supposed to be dependent on the parameters of the distribution of future returns. One can be tempted to define "overall risk aversion" as the sum of regret aversion and traditional risk aversion ($\rho + \lambda$). But hedging behavior will be different depending on the two components of overall risk aversion.

Our results could provide a framework to estimate agents' regret and risk aversion. We could think of a laboratory experiment where, for a given wealth position, we give investors different scenarios about expected currency movement and elasticity of asset return to currency movement. As ρ and λ enter the three hedging terms in a different fashion, we can derive estimates of both regret aversion and risk aversion.

Figure 3: Optimal hedge ratio as a function of regret aversion

The figure gives the optimal hedge ratio and the three hedging terms for regret, speculation and covariance motives as a function of regret aversion. Risk aversion is set at one, and regret aversion varies from zero to fifteen. The total hedge ratio is equal to 100% minus the three terms. The three panels use three different sets of market expectations for expected currency movement and covariance between asset return and currency. The currency standard deviation is set at 10% per year ($\Sigma_s = 1\%$)

Figure 3a: $\bar{s} = 0$ and $\mathrm{cov}(\tilde{r}, \tilde{s}) = 0$

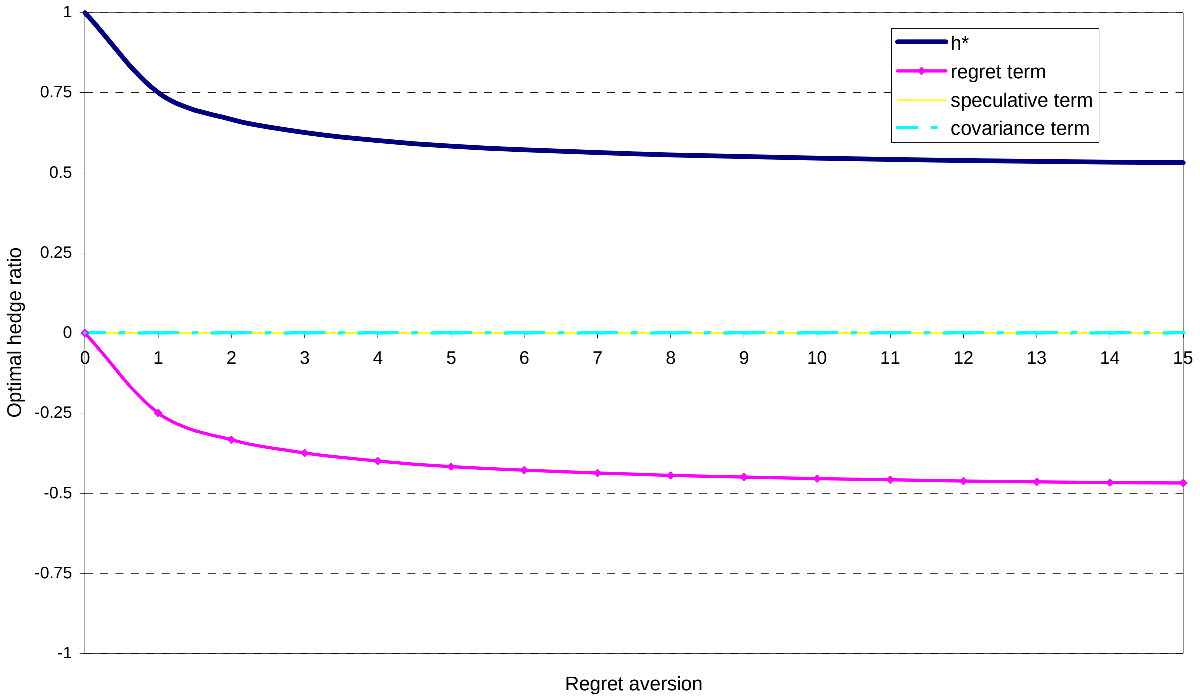


Figure 3b: $\bar{s} = 1\%$ and $\mathrm{cov}(\tilde{r}, \tilde{s}) = 0$

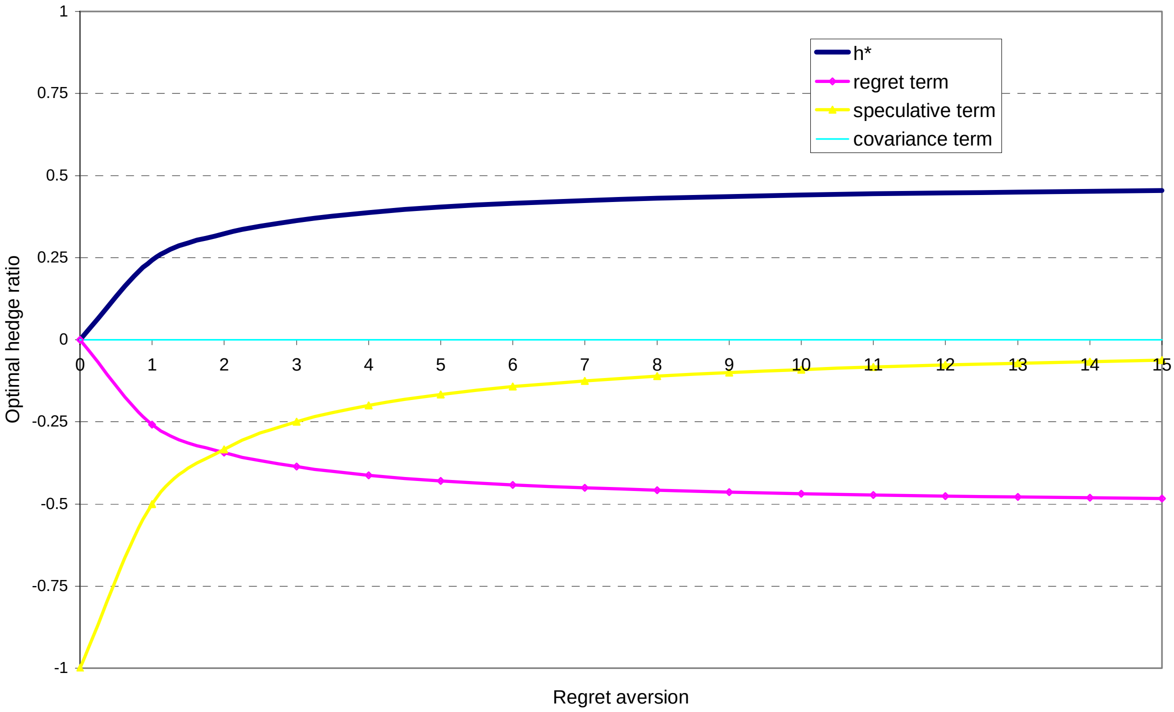
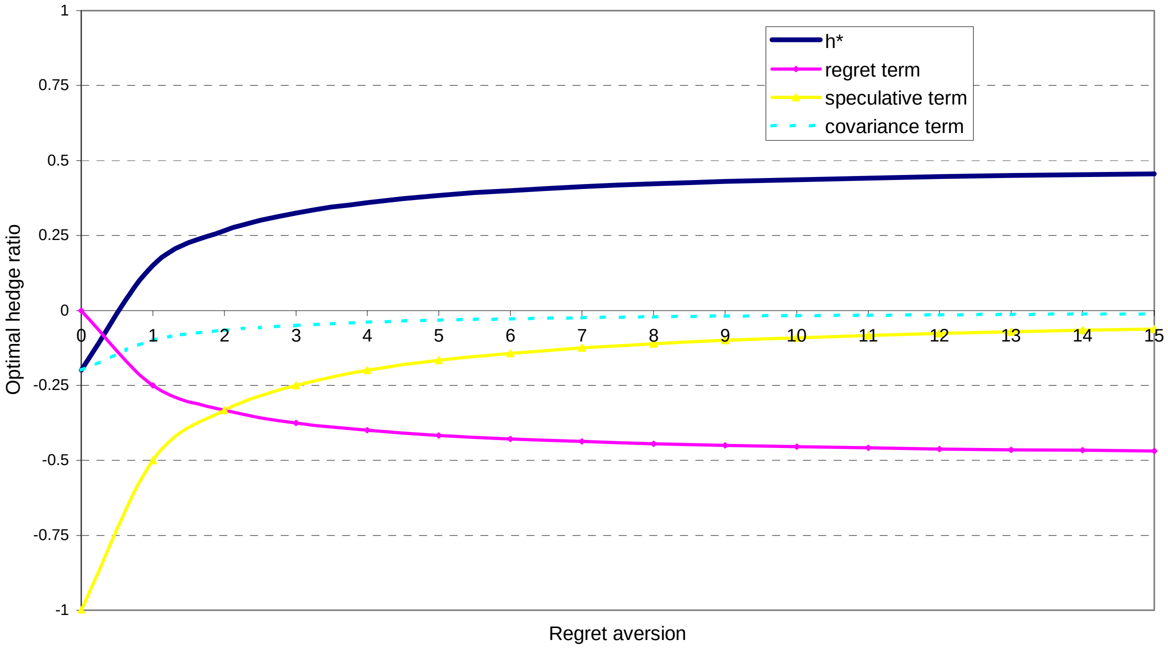


Figure 3c: $\bar{s} = 1\%$ and $\mathrm{cov}(\tilde{r}, \tilde{s}) / \Sigma_s = -0.2$



6. Conclusion

We present a model of optimal currency-hedging choices based on regret theory, a behavioral finance theory where investors reach optimal investment decision taking the expected pain of future regret into account. Regret averse investors derive utility from their global asset allocation (as a "traditional" expected-utility maximizer investor) but, in addition, they also experience regret for having chosen a currency exposure that proves, with hindsight, inappropriate. We have shown that investors who exhibit regret-theoretical preferences will tend to exhibit a wide diversity of hedging policies depending on their level of regret aversion.

In the absence of priors about expected currency movements and about correlation between asset returns and currency movements, all traditional expected-utility maximizers will fully hedge (100%) currency risk, whatever their (positive) level of risk aversion. However, a regret averse investor will be less prone to hedge because, ex-post, such a decision will be associated with the regret of having hedged too much in the case of a foreign currency appreciation. In general, regret averse investors will adopt a currency-risk-minimizing hedging strategy with a hedge ratio that depends on their level of regret aversion. The optimal hedge ratio lies between 50% (regret aversion dominates risk aversion) and 100% (no regret aversion).

In the general currency hedging case, i.e. when investors have beliefs about the future currency movements and when currency risk may be correlated with the return on the foreign assets, regret will induce regret averse investors to adopt hedging decisions that differ in many respects from those of traditional investors

Finally, we find that highly-regret-averse investors (regret dominates risk aversion) will neglect speculation motives and covariance effects and care only about regret. They will follow the widely observed 50% naive hedging policy, thus a robust prediction of our model whatever the beliefs in expected currency movements regret averse investors may have.

In summary, differences in the level of regret aversion among investors may explain why we observe such a wide diversity of currency hedging policies among institutional investors.

It should be stressed that regret aversion is not observationally equivalent to risk aversion. Our results could provide a framework to estimate agents' regret and risk aversion. This is left for further experimental research.

Regret averse investors could find currency options attractive tools to manage their regret risk exposure. Introducing options in the model is not an easy task. Pricing options itself could be difficult in a world where risk is two-dimensional (volatility and regret). This is also left for future research.

We believe the regret theoretic approach applied to currency hedging decisions bears some interesting normative implications as discussed along the course of this article. More generally, we believe that regret theory may be an interesting addition to the behavioral finance theory toolbox. It can be thought of as a normative complement to prospect theory that has seen widespread use in behavioral financial modeling.

Appendix A: Concavity of *u* with respect to *s*

Let's take derivatives with respect to *s* of *u*(*h,R,s*) in given in , where we replaced (1-*h*) by γ:

$$u(h,\tilde{R},\tilde{s}) = v(\tilde{R}+\gamma\tilde{s}) + f_{s+}(v(\tilde{R}+\gamma\tilde{s}) - v(\tilde{R}+\tilde{s})) + f_{s-}(v(\tilde{R}+\gamma\tilde{s}) - v(\tilde{R}))$$

The first derivative for positive values of s is:

$$\frac{\partial u}{\partial s} = \gamma v'(R+\gamma s) + f'(v(R+\gamma s) - v(R+s)) \times \left[\gamma v'(R+\gamma s) - v'(R+s)\right]$$

The second derivative for positive values of *s* is:

$$\begin{aligned}&\frac{\partial^2 u}{\partial s^2} = \gamma^2 v''(R+\gamma s) + f''(v(R+\gamma s) - v(R+s)) \times \left[\gamma v'(R+\gamma s) - v'(R+s)\right]^2 \\ &+ f'(v(R+\gamma s) - v(R+s)) \times \left[\gamma^2 v''(R+\gamma s) - v''(R+s)\right]\end{aligned} \tag{23}$$

For all values of *h* and *R*, the second derivative $\frac{\partial^2 u}{\partial s^2}$ is negative as $v',f'>0$ and $f'',v''''<0$.

The first derivative for negative values of s is:

$$\frac{\partial u}{\partial s} = \gamma v'(R+\gamma s) + f'(v(R+\gamma s) - v(R)) \times \left[\gamma v'(R+\gamma s)\right]$$

The second derivative for negative values of *s* is:

$$\begin{aligned}&\frac{\partial^2 u}{\partial s^2} = \gamma^2 v''(R+\gamma s) + f''(v(R+\gamma s) - v(R)) \times \left[\gamma v'(R+\gamma s)\right]^2 \\ &+ f'(v(R+\gamma s) - v(R)) \times \left[\gamma^2 v''(R+\gamma s)\right]\end{aligned} \tag{24}$$

For all values of *h* and *R*, the second derivative $\frac{\partial^2 u}{\partial s^2}$ is negative.

So the function *u*(.) is a continuous function, concave with respect to *s* for all *s*>0 and all *s*>0. Furthermore, in *s*=0 its derivative is larger on the left-hand side than on the right-hand side. So its right-hand (or left-hand) derivative is decreasing. As indicated in Royden (1988), proposition 18, chapter 5, this is a sufficient condition for *u* to be concave with respect to *s*.

Appendix B: Concavity of *Eu* with respect to *h*

We first show that *u*(.) is concave in *h*. Let's take derivatives with respect to *h* of *u*(*h,R,s*) in given in (5):

$$u(h,R,s)=v(R+\left[1-h\right]s)+f_{s+}(v(R+\left[1-h\right]s)-v(R+s))+f_{s-}(v(R+\left[1-h\right]s)-v(R))$$

The first derivative is:

$$\frac{\partial u}{\partial h}=-sv'-sf'\times v'$$

Where the argument of *v*'(.) is $R+\left[1-h\right]s$, and the argument of *f*'(.) is $v(R+\left[1-h\right]s)-v(R+s)$ if s is positive, and $v(R+\left[1-h\right]s)-v(R)$ if s is negative. So $\frac{\partial u}{\partial h}$ is always positive and continuous, even in *s*=0.

The second derivative is:

$$\frac{\partial^2 u}{\partial h^2}=s^2v''-s(-sf''\times v'^2-sf'v'')=s^2(v''(1+f')+f''v'^2) \qquad (25)$$

Where *v*'(.) and *v*''(.) are valued at $R+\left[1-h\right]s$ and *f*'(.) and *f*''(.) are valued at is $v(R+\left[1-h\right]s)-v(R+s)$ if s is positive, and $v(R+\left[1-h\right]s)-v(R)$ if s is negative.

For all values of *h*, *s* and *R*, the second derivative $\frac{\partial^2 u}{\partial h^2}$ is negative as $v',f'>0$ and $f'',v''''<0$.

Let's now turn to *Eu* which can be written as a function of *h*. The second derivative of *Eu* with respect to *h* is :

$$\frac{\partial^2 Eu}{\partial h^2}=E\frac{\partial^2 u}{\partial h^2}=Es^2(v''(1+f')+f''v'^2)$$

Because $\frac{\partial^2 u}{\partial h^2}$ is negative for all values of *h*, *R* and *s*, so is $\frac{\partial^2 Eu}{\partial h^2}$.

Appendix C: Derivation of the optimal hedge ratio

The Expected utility is given by:

$$Eu(h,R,s) = Ev(R+\left[1-h\right]s) + E_{s+} f(v(R+\left[1-h\right]s) - v(R+s))$$
$$+E_{s-} f(v(R+\left[1-h\right]s) - v(R))$$

The derivations can be made more formal by setting $s = \xi s'$, $R = \xi R'$ and letting ξ become very small, where s' and R'' are multivariate normal distributions. All covariances of s and R are of order ξ^2 and we neglect terms of order higher the 2. To simplify notations, we skip this straightforward step in the derivations presented below.

We develop the value function $v(.)$ around $\bar{R}$ and the regret function $f(.)$ around 0. So the implicit arguments is $\bar{R}$ for all derivatives of v, and 0 for all derivatives of $f(.)$. With the additional notations Σ_r and $\mathrm{cov}(r,s)$ for the variance of r and the covariance between r and s:

$$Ev(R+\left[1-h\right]s) \approx v(\bar{R}) + \left[1-h\right]\bar{s}v' + \frac{1}{2}\left[\Sigma_r + 2(1-h)\mathrm{cov}(r,s) + (1-h)^2\Sigma_s\right]v'' \tag{26}$$

Over $s+$:

$$\begin{aligned} &E_{s+} f(v(R+\left[1-h\right]s) - v(R+s)) \approx E_{s+}\left[v(R+\left[1-h\right]s) - v(R+s)\right] f' \\ &+\frac{1}{2}E_{s+}\left[v(R+\left[1-h\right]s) - v(R+s)\right]^2 f'' \end{aligned} \tag{27}$$

with:

$$E_{s+}(v\left(R+\left[1-h\right]s\right) - v(R+s)) \approx -h\bar{s}_+ v' - h\mathrm{cov}_{s+}(r,s)v'' + \frac{1}{2}(h^2-2h)\Sigma_{s+}v''$$

$$E_{s+}\left[v(R+\left[1-h\right]s) - v(R+s)\right]^2 \approx h^2\Sigma_{s+}v'^2$$

Hence (27) becomes:

$$E_{s+} f \approx \left[-h\bar{s}_+ v' - h\mathrm{cov}_{s+}(r,s)v'' + \frac{1}{2}(h^2-2h)\Sigma_{s+}v''\right] f' + \frac{1}{2}h^2\Sigma_{s+}v'^2 f'' \tag{28}$$

Over s-:

$$\begin{aligned} &E_{s-} f(v(R+\left[1-h\right]s) - v(R)) \approx E_{s-}\left[v(R+\left[1-h\right]s) - v(R)\right] f' \\ &+\frac{1}{2}E_{s-}\left[v(R+\left[1-h\right]s) - v(R)\right]^2 f'' \end{aligned} \tag{29}$$

with:

$$E_{s-}(v\left(R+\left[1-h\right]s\right) - v(R)) \approx (1-h)\bar{s}_- v' + (1-h)\mathrm{cov}_{s-}(r,s)v'' + \frac{1}{2}(1-h)^2\Sigma_{s-}v''$$

$$E_{s-}\left[v(R+\left[1-h\right]s) - v(R)\right]^2 \approx (1-h)^2\Sigma_{s-}v'^2$$

Hence (29) becomes:

$$E_{s-} f \approx \left[(1-h)\overline{s}_{-} v' + (1-h)\operatorname{cov}_{s-}(r,s) v'' + \frac{1}{2}(1-h)^2 \Sigma_{s-} v'' \right] f' + \frac{1}{2}(1-h)^2 \Sigma_{s-} v'^2 \; f'' \qquad (30)$$

The expected utility is the sum of three terms:

Eu = (26) +(28) + (30)

Remember that $\overline{s} = \overline{s}_{+} + \overline{s}_{-}$, $\Sigma_s = \Sigma_{s+} + \Sigma_{s-}$ and $\operatorname{cov}(r,s) = \operatorname{cov}_{s+}(r,s) + \operatorname{cov}_{s-}(r,s)$. Then:

$$\begin{aligned}
&Eu \approx v(\overline{R}) + (1-h) v' \overline{s} + \frac{1}{2}(1-h)^2 v'' \Sigma_s + \frac{1}{2}\Sigma_R v'' + (1-h) v'' \operatorname{cov}(r,s) \\
&f'(v'\overline{s}_{-} + v'' \operatorname{cov}_{s-}(r,s)) - h f'(v'\overline{s} + v'' \operatorname{cov}(r,s)) \\
&+ \frac{1}{2} h^2 v'' f' \Sigma_s - h v'' f' \Sigma_s + \frac{1}{2} v'' f' \Sigma_{s-} \\
&+ \frac{1}{2} h^2 v'^2 \; f'' \Sigma_s - h v'^2 \; f'' \Sigma_{s-} + \frac{1}{2} v'^2 \; f'' \Sigma
\end{aligned}$$

Let's compute the optimal hedge ratio by setting to zero the derivative of *Eu* with respect to *h*. This the first order condition without constraints on *h*.

$$\begin{aligned}
&0 = -\left[v'\overline{s} + v''\Sigma_s + v'' \operatorname{cov}(r,s) + f'(v'\overline{s} + v'' \operatorname{cov}(r,s)) + v'' f' \Sigma_s + v'^2 \; f'' \Sigma_{s-} \right] \\
&+ h\left[v'' \Sigma_s + v'' f' \Sigma_s + v'^2 \; f'' \Sigma_s \right]
\end{aligned}$$

$$h^* = \frac{v'\overline{s} + v''\Sigma_s + v'' \operatorname{cov}(r,s) + f'(v'\overline{s} + v'' \operatorname{cov}(r,s)) + v'' f' \Sigma_s + v'^2 \; f'' \Sigma_{s-}}{v''\Sigma_s + v'' f' \Sigma_s + v'^2 \; f'' \Sigma_s}$$

$$\begin{aligned}
&h^* = 1 - \frac{\Sigma_{s+}}{\Sigma_s} \times \frac{v'^2 \; f''}{v''(1+f') + v'^2 \; f''} \\
&+ \frac{\overline{s}}{\Sigma_s} \times \frac{v'(1+f')}{v''(1+f') + v'^2 \; f''} + \frac{\operatorname{cov}(r,s)}{\Sigma_s} \times \frac{v''(1+f')}{v''(1+f') + v'^2 \; f''}
\end{aligned} \qquad (31)$$

Replacing risk aversion $\lambda = -v''/v'$ and regret aversion $\rho = \frac{-v' f''}{1+f'}$, we get:

$$h^* = 1 - \frac{\Sigma_{s+}}{\Sigma_s} \times \frac{\rho}{\rho+\lambda} - \frac{\overline{s}}{\Sigma_s} \times \frac{1}{\rho+\lambda} + \frac{\operatorname{cov}(r,s)}{\Sigma_s} \times \frac{\lambda}{\lambda+\rho} \qquad (32)$$

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